Stochastic performance modelling (2WB60)

Instruction week 6, 18 March 2016

Exercise 1. A shopkeeper keeps track of the amount of money X he earns per day. The shopkeeper reports an average earning of $\mathbb{E}[X] = 1500$ per day. He also tells us that $\mathbb{E}[X^2] = 2.41 \cdot 10^6$. What type of approximating distribution would you fit for the distribution of X and what are its parameters?

Exercise 2. Verify Example 6.2.1 of the lecture notes.

Exercise 3. Consider a single processor which handles three types of tasks. Type-i tasks arrive according to a Poisson process with rate λ_i and the service requirements of type-i tasks are exponentially distributed with mean $1/\mu_i$, $i = 1, 2, 3$, with $\rho = \lambda_1/\mu_1 + \lambda_2/\mu_2 + \lambda_3/\mu_3 < 1$. Tasks are served in order of arrival.

(a) Argue why the system may be viewed as an $M/H_3/1$ queue, where H_3 represents a hyperexponential service requirement distribution.

(b) Determine the mean waiting time of type-1 tasks. (**A**: $\mathbb{E}[W_1] = (1-\rho)^{-1} \sum_{i=1}^{3} \rho_i/\mu_i$)

(c) Suppose that at some point in time there are at least three tasks present in the system. What is the probability that the first two waiting tasks (thus excluding the task in service) belong to different types?

(d) Now suppose that the tasks are served according to a preemptive-resume priority strategy, which assigns the highest priority to type-1 tasks, the next highest priority to type-2 tasks, and the lowest priority to type-3 tasks. Determine the mean waiting time of a type-i task.

Exercise 4. A machine mounts electronic components on three different types of printed circuit boards, type A, B and C boards say. On average 54 type-A boards arrive per hour, 48 type-B boards, and 18 type-C boards. The arrival processes are Poisson. The mounting times are exactly 20 seconds for type A, 30 seconds for type B, and 40 seconds for type C. The boards are processed in order of arrival.

(a) Calculate the mean overall waiting time $\mathbb{E}[W]$. (**A**: $\mathbb{E}[W] = 13/6$)

(b) Calculate for each type of printed circuit board the mean sojourn time and also calculate the mean overall sojourn time. (**A**: $\mathbb{E}[S_A] = 15/6$, $\mathbb{E}[S_B] = 16/6$, $\mathbb{E}[S_C] = 17/6$, $\mathbb{E}[S] = 157/60$)

From now on, suppose that there are priority rules in effect. Type-A boards have the highest priority, type-B boards have intermediate priority, and type-C boards have the lowest priority. The priorities are non-preemptive.

(c) Compute $\mathbb{E}[W_A]$ and $\mathbb{E}[W_B]$, the mean waiting times of type-A and type-B boards, respectively. (**A**: $\mathbb{E}[W_A] = 13/42$, $\mathbb{E}[W_B] = 65/63$)

(d) Use the work conservation law in combination with your answers to questions (a) and (c) to calculate $\mathbb{E}[W_C]$, the mean waiting time of type-C boards. (**A**: $\mathbb{E}[W_C] = 65/9$)

(e) Calculate for each type of printed circuit board the mean sojourn time. (**A**: $\mathbb{E}[S_A] = 27/42$, $\mathbb{E}[S_B] = 193/126$, $\mathbb{E}[S_C] = 71/9$)