Exercise 3 Stochastic Models of Manufacturing Systems 4T400, 6 May

1. Each week a very popular lottery in Andorra prints 10^4 tickets. Each tickets has two 4-digit numbers on it, one visible and the other covered. The numbers are randomly distributed over the tickets. If someone, after uncovering the hidden number, finds two identical numbers, he wins a large amount of money. What is the average number of winners per week?

2. Two people, strangers to one another, both living in Eindhoven, meet each other. Each has approximately 200 acquaintances in Eindhoven. What is the probability of the two people having an acquaintance in common?

3. X is the distance to 0 of a random point in a disk of radius r.

   - What is the density of X?
   - Calculate $E(X)$ and $\text{var}(X)$.

4. Let X be random on $(0, 1)$ and Y be random on $(0, X)$.

   - What is the density of the area of the rectangle with sides X and Y?
   - Calculate the expected value of the area of the rectangle with sides X and Y.

5. A batch consists of n items with probability $(1 - p)p^{n-1}, n \geq 1$. The production time of a single item is uniform between 4 and 10 minutes. What is the mean production time of a batch?

6. Let X have a mixed binomial distribution: with probability 0.5, the random variable X is binomial with parameters $n = 20$ and $p = 0.1$, and otherwise, X is binomial with parameters $n = 20$ and $p = 0.9$. Let $X_i, i = 1, 2, \ldots$ be independent random variables, with the same distribution as X, and consider the sum $S_n = X_1 + \ldots + X_n$. For each $n = 1, 5, 10, 20$, generate many samples of S_n (for example, by using χ) and plot a histogram (for example in **R**). What is your conclusion?

7. Consider a two-machine production line producing fluid. The production rate of machine 1 is 5, the rate of machine 2 is 4. Machine 2 never fails, but machine 1 is subject to breakdowns. The up times and down times have been collected during a long period. The size of the fluid buffer is K.

   - Fit some "simple" distributions to the sample mean and sample variance of the up and down times, and graphically compare the fitted and the empirical distributions.
   - Estimate, by simulation, the throughput for $K = 20$ over a long simulation horizon T, say $T = 10^4$ time units. Repeat this many (say 10^3) times, and plot a histogram of the estimates for the throughput. What is your conclusion about the distribution of these estimates and how can this be explained?
   - Estimate the long-run throughput of the production line for various values of the buffer size K, by using both the empirical and fitted distributions. What are your conclusions?