Analysis of Manufacturing Systems 4AB00

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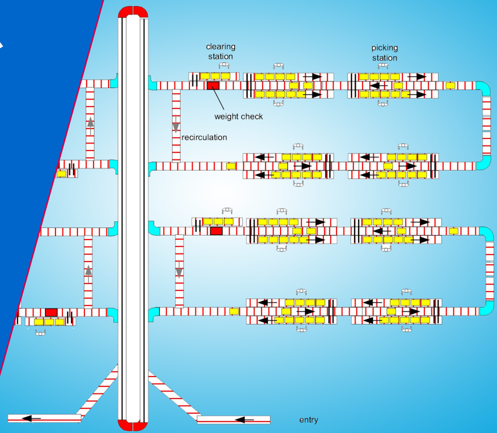

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Recap

A random variable X is continuously distributed if

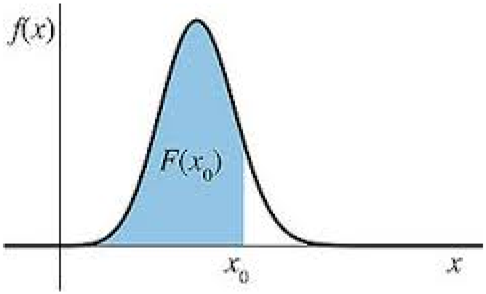
$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)dy,$$

where the function $f(x)$ satisfies

$$f(x) \geq 0 \text{ for all } x, \quad \int_{-\infty}^{\infty} f(x)dx = 1.$$

$F(x)$ is probability distribution of X and $f(x)$ is probability density of X.

Interpretation of density: $P(x < X \leq x + dx) \approx f(x)dx$



Recap

- Expected value of X with density $f(x)$ is

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

- Variance of X is

$$\begin{aligned} \text{var}(X) &= E((X - E(X))^2) \\ &= \int_{-\infty}^{\infty} (x - E(X))^2 f(x) dx \end{aligned}$$

Recap

- Uniform random variable X on (a, b),

$$f(x) = \frac{1}{b-a}, \quad a < x < b,$$

and $f(x) = 0$ otherwise. Then

$$P(X \le t) = \frac{t-a}{b-a}, t > 0, \quad E(X) = \frac{1}{2}(a+b), \quad \text{var}(X) = \frac{1}{12}(b-a)^2.$$

- Exponential random variable X with parameter (or rate) $\lambda > 0$,

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0,$$

and $f(x) = 0$ otherwise. Then

$$P(X \le t) = 1 - e^{-\lambda t}, t > 0, \quad E(X) = \frac{1}{\lambda}, \quad \text{var}(X) = \frac{1}{\lambda^2}.$$

Recap

- Memoryless property for exponential X: for all $t, s > 0$,

$$P(X > t + s | X > s) = P(X > t).$$

- For independent exponentials $X_1, \ldots, X_n$ with rates $\lambda_1, \ldots, \lambda_n$,

$$P(\min_i X_i > t) = e^{-(\lambda_1 + \cdots + \lambda_n)t}, \quad t > 0.$$

So $\min\{X_1, \ldots, X_n\}$ is exponential with rate $\lambda_1 + \cdots + \lambda_n$.

Recap

Example:

Consider system with two components A and B. The lifetime of component A and B are independent and exponential. The mean lifetime of A is 20 hours, the mean lifetime of B is 40 hours. At $t = 0$ both components are working.

Questions:

- What is the probability that at $t = 20$ hours, both components still work?
- What is the mean time till the first component breaks down?

Recap

Example:

Consider system of n components where life time of component i is exponential with parameter λ. Let X denote time till all n components failed. Then

$$E(X) = \frac{1}{n\lambda} + \frac{1}{(n-1)\lambda} + \cdots + \frac{1}{\lambda} = \sum_{i=1}^{n} \frac{1}{i\lambda}.$$

Recap

Manufacturing network

- Network of workstations W_i, $i = 1, \ldots, N$, with m_i identical machines
- Mean processing time $t_{s,i}$ in workstation W_i
- External inflow λ_i jobs per unit time unit
- Fraction p_{ij} of throughput δ_i of workstation W_i diverted to W_j

Then

- Throughput δ_i of workstation W_i satisfies conservation of flow

$$\delta_i = \lambda_i + \sum_{j=1}^{N} \delta_j p_{ji}, \quad i = 1, \ldots, N$$

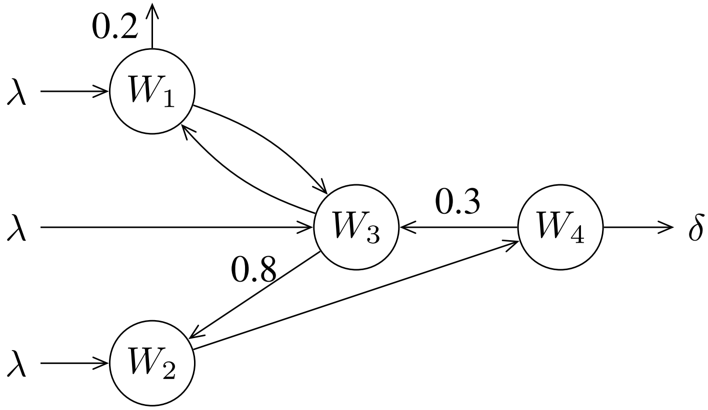
- Utilization u_i of machine in workstation W_i is

$$u_i = \frac{\delta_i t_{s,i}}{m_i} < 1$$

Bottleneck W_b is the one with maximal utilization

Recap

Example:



Workstation	m	t_s [hour]
W_1	1	5.0
W_2	1	2.5
W_3	1	3.0
W_4	1	2.0

Recap

Example:

- Formulate the flow equations for δ_i.
- Express output rate δ_i of workstation i as function of λ.
- Express utilization u_i of workstation i as function of λ.
- What is the fraction of lots that is scrapped?
- What is the bottleneck workstation?
- What is maximal outflow rate (or maximal throughput) $\delta_{\max}$?

Recap

Little's Law:

Consider a stable system

- w is the mean WIP level in the system
- δ is the throughput of the system
- φ is the mean flow time in the system

Then

$$w = \delta\varphi$$

Variability

Little's Law:

$$\delta = \frac{w}{\varphi}$$

Same throughput can be achieved with

- large WIP w and long flowtimes φ
- small WIP w and short flowtimes φ

Question: What causes the difference?

Answer: Variability!

Variability

Controllable variation: Result of decisions

- Variability in products produced by the plant
- Batch movement of material (first finished part waits longer than last one)

Random variation: Result of events beyond our control

- Time between customer demands
- Machine failures

Variability

First moment intuition (mean):

- Get more products out by speeding up bottleneck machine

Second moment intuition (variance):

- Which is more variable: time to process individual part or a batch?
- Which results in greater improvement of line performance:
  - Reduce variability of process times closer to raw materials?
  - Reduce variability of process times closer to customers?
- Which are more disruptive:
  - Short frequent machine failures?
  - Long infrequent machine failures?

Process time variability

- Effective process time is total time seen by a job at a station, includes
  - natural process time
  - setups
  - rework
  - operator unavailability
  - and other shop floor realities
- Standard deviation σ is absolute measure of variability
- Coefficient of variation c is relative measure of variability

$$c = \frac{\sigma}{t}$$

where t is the mean and σ the standard deviation

Process time variability

Sources of variability

- Natural variability (differences in operators, machines, material)
- Random outages (failures)
- Setups
- Operator unavailability
- Rework

Classes of variability

- Low $c < 0.75$: process times without outages
- Moderate $0.75 \leq c < 1.33$: process times with short outages (setups)
- High $c \geq 1.33$: process times with long outages (failures)

Natural variability

Catch-all category due to differences in

- operators
- machines
- composition in material

Natural coefficient of variation

$$c_0 = \frac{\sigma_0}{t_0}$$

where t_0 and σ_0 are mean and standard deviation of natural process time

Natural process times typically have low variability: $c_0 < 0.75$

Preemptive breakdowns

Availability (fraction of time machine is available)

$$A = \frac{m_f}{m_f + m_r}$$

where

- m_f is mean time to failure
- m_r is mean time to repair

Adjusting natural process time t_0 to effective process time

$$t_e = \frac{t_0}{A}$$

and effective capacity of workstation with m machines

$$r_e = \frac{m}{t_e} = A\frac{m}{t_0} = Ar_0$$

Preemptive breakdowns

Example: (time unit is minute)

Machine M_1: $t_0 = 15$, $\sigma_0 = 3.35$, $c_0 = 0.223$, $m_f = 744$, $m_r = 248$

Machine M_2: $t_0 = 15$, $\sigma_0 = 3.35$, $c_0 = 0.223$, $m_f = 114$, $m_r = 38$

Then for both machines

$$A = 0.75, \quad t_e = 20$$

So both have effective capacity

Preemptive breakdowns

Assumption: Time to failure is exponential

$$
\begin{aligned}
t_e &= \frac{t_0}{A} \\
\sigma_e^2 &= \left(\frac{\sigma_0}{A}\right)^2 + \frac{(m_r^2 + \sigma_r^2)(1-A)t_0}{A m_r} \\
c_e^2 &= \frac{\sigma_e^2}{t_e^2} = c_0^2 + (1 + c_r^2)A(1-A)\frac{m_r}{t_0} \\
&= c_0^2 + A(1-A)\frac{m_r}{t_0} + c_r^2 A(1-A)\frac{m_r}{t_0}
\end{aligned}
$$

where σ_r is standard deviation of time to repair

Note: c_e^2 increases in m_r, so long repair times induce more variability than short ones

Preemptive breakdowns

Example: (including variability effects)

Machine M_1: $t_0 = 15$, $\sigma_0 = 3.35$, $c_0 = 0.223$, $m_f = 744$, $m_r = 248$, $c_r = 1$

Machine M_2: $t_0 = 15$, $\sigma_0 = 3.35$, $c_0 = 0.223$, $m_f = 114$, $m_r = 38$, $c_r = 1$

Then for M_1

$$c_e^2 = 6.25$$

and for M_2

$$c_e^2 = 1$$

So machine M_1 exhibits much more variability than M_2!

Nonpreemptive outages

Controllable downtimes such as

- tool changes
- setups
- preventive maintenance
- shift changes

Example:

Machine needs setup with mean t_s and coefficient of variation c_s after having produced on average N_s jobs. Then

$$\begin{aligned} t_e &= t_0 + \frac{t_s}{N_s} \\ \sigma_e^2 &= \sigma_0^2 + \frac{\sigma_s^2}{N_s} + \frac{N_s - 1}{N_s} t_s^2 \\ c_e^2 &= \frac{\sigma_e^2}{t_e^2} \end{aligned}$$

Nonpreemptive outages

Example: (time unit is hour)

Machine M_1 is flexible, no setups: $t_0 = 1.2$, $c_0 = 0.5$

Machine M_2 is fast, with setups: $t_0 = 1$, $c_0 = 0.25$, $N_s = 10$, $t_s = 2$, $c_s = 0.25$

Then for M_1 and M_2

$$t_e = 1.2$$

so same effective capacity. Which has less variability? For M_1

$$c_e^2 = c_0^2 = 0.25$$

and for M_2

$$c_e^2 = 0.31$$

So flexible machine M_1 exhibits less variability than M_2!

Rework

- Workstation performs tasks
- Mean task time is t_0 with standard deviation σ_0
- Task is checked after completion
- Task is done correctly with probability q
- If not, task is repeated (and checked till it is eventually correct)

Then each task is repeated on average

$$N_r = \frac{1}{q}$$

and

$$\begin{aligned} t_e &= N_r t_0 \\ \sigma_e^2 &= N_r \sigma_0^2 + N_r(N_r - 1)t_0^2 \\ c_e^2 &= \frac{c_0^2}{N_r} + \frac{N_r - 1}{N_r} \end{aligned}$$