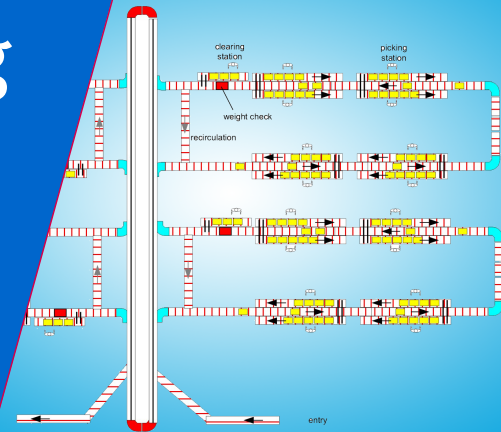


# Analysis of Manufacturing Systems 4AB00

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## Lecture notes on Analysis of Manufacturing Systems

- Chapter 3
- Chapter 6
- Chapter 7

Final examination: **Open book**

# Recap: Variability interactions

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- Inter-arrival time is **general** with mean  $t_a$  and standard deviation  $\sigma_a$
- Process time is **general** with mean  $t_e$  and standard deviation  $\sigma_e$
- **$m$  parallel identical machines**

Then

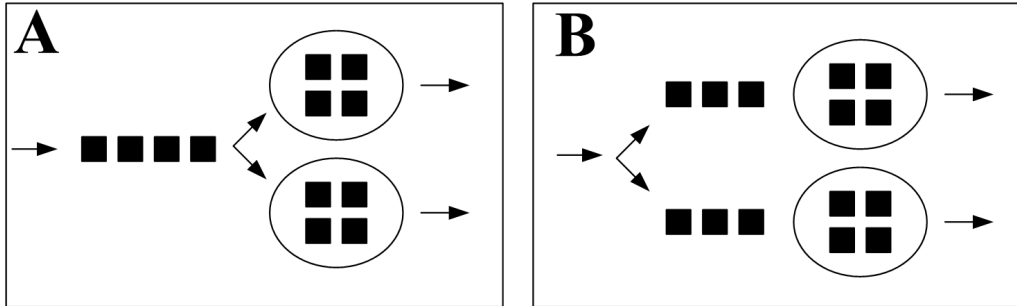
$$\varphi \approx \gamma \cdot \frac{u\sqrt{2(m+1)}-1}{1-u} \cdot \frac{t_e}{m} + t_e$$

with

- **$u = t_e/(m \cdot t_a)$**  (utilization per machine)
- $\gamma = \frac{1}{2} \cdot (c_a^2 + c_e^2)$
- $c_a = \sigma_a/t_a$
- $c_e = \sigma_e/t_e$

# Example: Two configurations

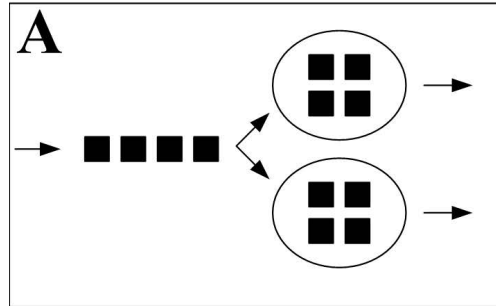
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- 2 parallel batch machines
- Batch size is  $k = 4$
- Jobs arrive according to Poisson stream with rate  $r_a = 1$  jobs per hour
- Mean batch processing time is  $t_e = 6$  hours
- Variance is  $\sigma_e^2 = 18 \text{ hours}^2$

# Example: Configuration A

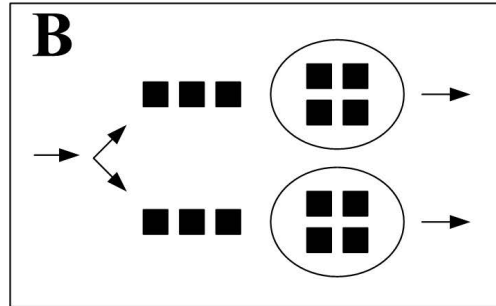
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Single common buffer

Questions:

- What is utilization of each machine?
- Determine mean flow time  $\varphi$

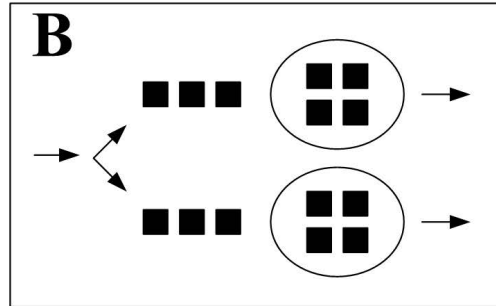


Each machine with **own local buffer** and **round robin assignment**

So job 1, 3, ... is sent to buffer 1, and job 2, 4, ... to buffer 2

## Questions:

- What is utilization of each machine?
- Determine mean flow time  $\varphi$
- Compare  $\varphi$  with Configuration A: Explanation?



Each machine with **own local buffer** and **random assignment**

So each lot is sent to buffer 1 with probability  $\frac{1}{2}$  and to buffer 2 otherwise

## Questions:

- What is utilization of each machine?
- Determine mean flow time  $\varphi$
- Compare  $\varphi$  with Configuration B with round robin: Explanation?

- Jobs arrive according to Poisson stream with rate  $r_a = 0.75$  jobs per hour
- Jobs are processed by single machine, one by one
- Process time of jobs is constant,  $t_e = 1$  hour
- Probability of rework is  $p$

## Questions:

- What is mean and cv of **effective process time**?
- For which  $p$  is system stable?
- Set  $p = 0.2$  and estimate mean flow time in buffer  $\varphi_B$
- What is effect of reducing  $p = 0.2$  to  $p = 0.18$ ? Explain!



## Types of batches:

- **Process** batch: Many parts processed together
  - Sequential:  
Parts are produced sequentially before change-over to other family
  - Simultaneous:  
Parts are produced simultaneously (e.g., in furnace)
- **Transfer** batch: Many parts are moved together
  - The smaller the batch the less time waiting to form the batch
  - The smaller the batch the more material handling

## Model:

- Two-machine serial line
- Single parts arrive with rate  $r_a$  at machine 1
- Machine 1 processes parts one-by-one
- Processed parts are transferred to machine 2 in batches
- Transfer batch size is  $k$
- Machine 2 processes parts one-by-one
- $t_e(i)$  and  $c_e(i)$  are mean and cv of process time of machine  $i$

## Question:

How does mean flow time depend on batch size  $k$ ?

- $u(i) = r_a t_e(i)$  is utilization of machine  $i$
- $c_a(1)$  and  $c_d(1)$  are cv of arrivals and departures of parts at machine 1

$$c_a(1) = c_a$$

$$c_d^2(1) = u^2(1)c_e^2(1) + (1 - u^2(1))c_a^2(1)$$

- Mean flow time at machine 1

$$\varphi(1) = \left( \frac{c_a^2(1) + c_e^2(1)}{2} \right) \left( \frac{u(1)}{1 - u(1)} \right) t_e(1) + t_e(1)$$

- Mean wait-to-batch time at machine 1

$$\frac{k-1}{2r_a} = \frac{k-1}{2u(1)} t_e(1)$$

- Mean flow time at machine 2 (note  $c_d^2(2) = c_d^2(1)$ )

$$\begin{aligned}\varphi(2) &= \left( \frac{c_d^2(1)/k + c_e^2(2)/k}{2} \right) \left( \frac{u(2)}{1-u(2)} \right) k t_e(2) + \frac{k+1}{2} t_e(2) \\ &= \left( \frac{c_d^2(1) + c_e^2(2)}{2} \right) \left( \frac{u(2)}{1-u(2)} \right) t_e(2) + \frac{k+1}{2} t_e(2)\end{aligned}$$

- Mean flow time in line

$$\varphi(1) + \frac{k-1}{2u(1)} t_e(1) + \varphi(2)$$

## Observations:

- Total flow time increases proportionally with batch size
- This increase has **nothing** to do with process or arrival variability
- It is caused by variability due to **bad control**
- Impact of transfer batching is largest when utilization  $u(1)$  is low

Cycle (or flow) time  $T$  is made up of:

- Move time
- Queue time
- Setup time
- Process time
- Wait-to-batch time
- Wait-in-batch time

Manufacturing lead time  $l$  is:

- Time allowed to finish an individual job from start to end of the line

Service  $s$  is:

$$s = P(T \leq l) = F(l)$$

where  $F$  is distribution function of  $T$

If cycle times  $T$  are normally distributed, then for service  $s$

$$l = E(T) + z_s \sigma(T)$$

where  $z_s$  is the value for which  $\Phi(z_s) = s$

## Example:

- Cycle time has mean of 10 days and standard deviation of 3 days
- $z_{0.95} = 1.645$
- Then required manufacturing lead time for service  $s = 0.95$  percent

$$l = 10 + 1.645 \cdot 3 = 14.94 \approx 15 \text{ days}$$

- Above additional 5 days are called **safety lead time**

Performance is closely related to amount of buffers in the system:

- A **perfect** system will have no buffers at all
- A **poorly performing** system will contain large buffers

This implies that a manufacturing system is **lean** if:

- It reaches its objective (e.g. target throughput) with **minimal buffering cost**



## Increasing throughput:

Throughput TH of line is

$$TH = \text{bottleneck utilization} \times \text{bottleneck rate}$$

Checklist:

- **Increase bottleneck rate:**  
Add equipment, add staff, improve quality, and so on
- **Increase bottleneck utilization:**  
Reduce blocking and starvation by buffering
  - **WIP** by increasing buffers (immediately) in front and after bottleneck
  - **Capacity** by increasing rates of (highest utilization) **non-bottlenecks**

## Reducing cycle time:

Cycle time includes

- Queue time
- Process (wait-to and wait-in) batch time

Checklist:

Queue time is caused by utilization and variability

- **Reduce utilization of bottleneck:**  
Add equipment, reduces setups, decrease repairs, reduce flow, and so on
- **Reduce variability in process times and arrivals:**  
Reduce repair and setup times, improve quality, and so on

Process batch time is driven by process batch size

- **Batching optimization**
- **Setup reduction**

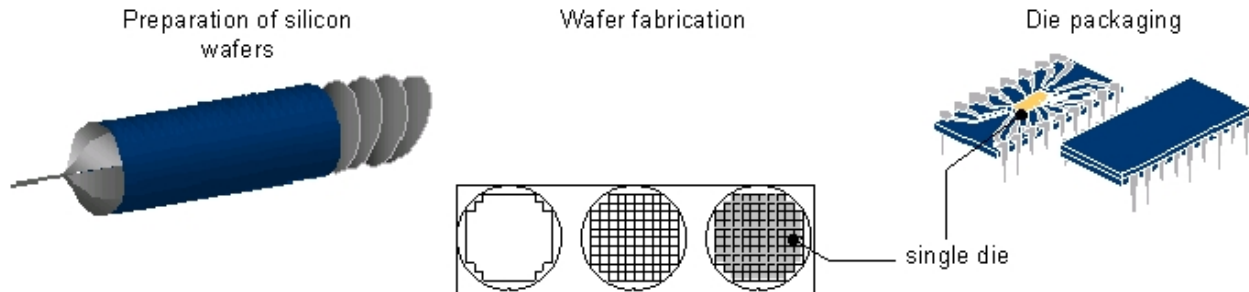


Overview in a waferfab [NXP, Crolles, Grenoble].

- Most electronic devices contain chips
- Chips contain Integrated Circuits (ICs)
- ICs are made on wafers: complex manufacturing process

# Example: Enhance throughput

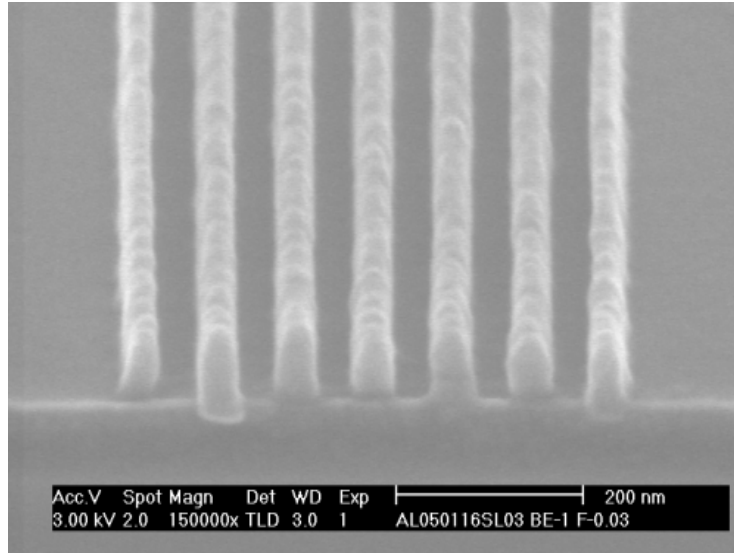
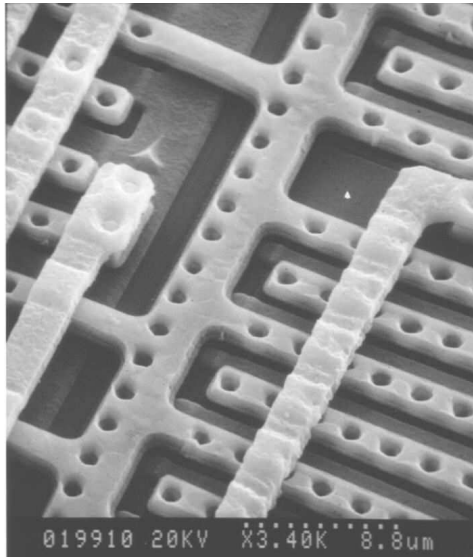
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- Raw silicon is extracted from sand
- Silicon is sliced into wafers
- Wafers are processed to create dies
- Dies are sawn out and packaged

# Example: Enhance throughput

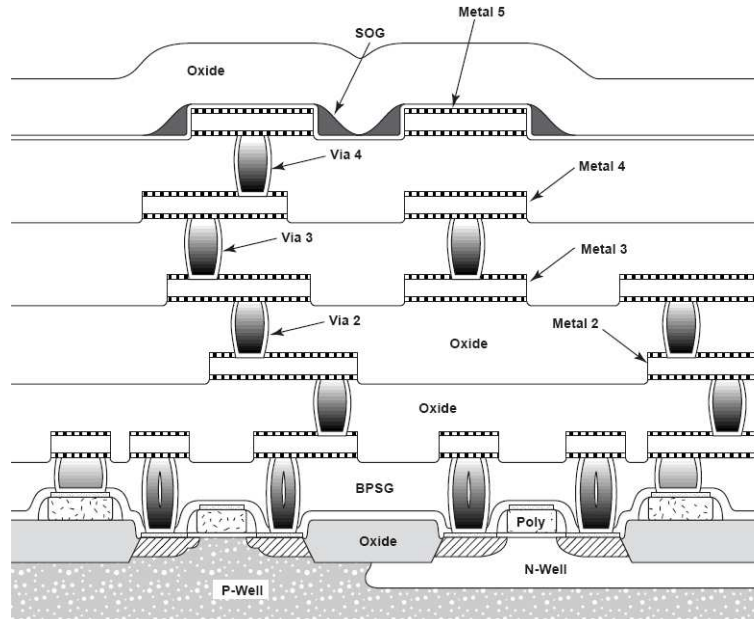
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Scanning electron microscope image of a wafer [AcceleratedAnalysis, IMEC].

# Example: Enhance throughput

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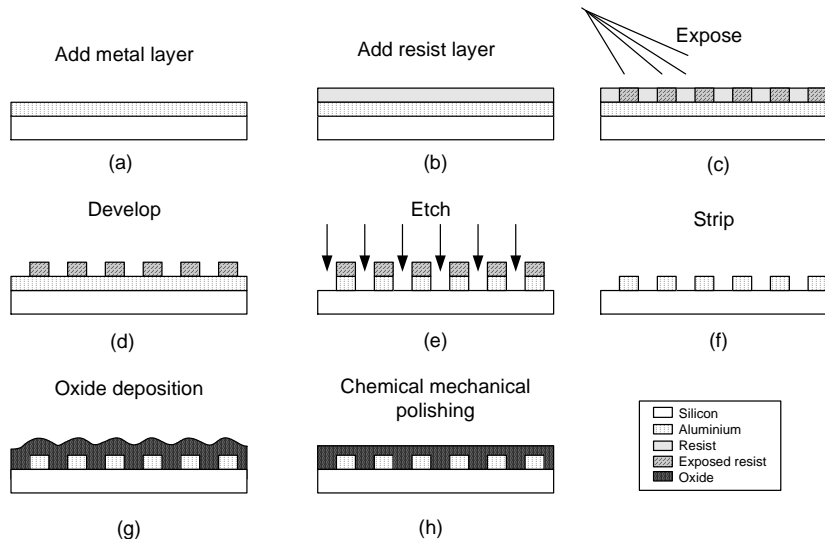


Cross section of a wafer

- Five metal layers (contains interconnections, lines and vias)
- Zero layer (contains functionality)

# Example: Enhance throughput

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- Wafer processing takes a few hundred of process steps
- Typical processes: deposition, exposure, etching, stripping, and oxidation
- Wafer fabrication shows re-entrant behavior:  
A pod contains some 10 wafers and revisits the various process areas

A printed-circuit board line with two stations:

- Station 1 (resist apply) applies photoresist material to circuit boards
- Station 2 (expose) exposes boards to ultraviolet light
- Space for WIP between two stations is 20 jobs
- Mean process time  $t_0(2) = 22$  (min),  $cv\ c_0(2) = 1$
- Mean process time  $t_0(1) = 19$  (min),  $cv\ c_0(1) = 0.5$
- $m_f(2) = 3\frac{1}{3}$  (hour),  $m_r(2) = 10$  (min)
- $m_f(1) = 48$  (hour),  $m_r(1) = 8$  (hour)
- Arrivals  $cv\ c_a = 1$
- **Desired throughput** is 2.4 jobs per hour

**Question:** How to achieve target throughput?



# Example: Enhance throughput

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Assume unlimited buffer between stations and  $r_a = 2.4$  jobs per hour:

$$\varphi_q(1) = 645(\text{min})$$

$$\varphi_q(2) = 887(\text{min})$$

so by Little's law

$$w_q(1) = 25.8$$

$$w_q(2) = 35.5 > 20!$$

Hence problem is rooted in long queue at station expose!

$$\begin{aligned}\varphi_q(2) &= \left( \frac{c_a^2(2) + c_e^2(2)}{2} \right) \left( \frac{u(2)}{1 - u(2)} \right) t_e(2) \\ &= (3.116)(12.15)(23.1)(\text{min}) \\ &= 887(\text{min})\end{aligned}$$

Effective process time

$$\begin{aligned}t_e(2) &= \frac{t_0(2)}{A(2)} \\ &= \frac{t_0(2)(m_f(2) + m_r(2))}{m_f(2)} \\ &= \frac{22(31/3 + 1/6)}{31/3} \\ &= 23.1(\text{min})\end{aligned}$$

so little improvement possible by **increasing availability**  $A(2)$

**Decreasing**  $u(2)$  is expensive option

Variability inflation factor

$$c_e^2(2) = 1.04, \quad c_a^2(2) = 5.27$$

so arrival variability is **dominant source of variability**

$$\begin{aligned} c_a^2(2) &= c_d^2(1) \\ &= u^2(1)c_e^2(1) + (1 - u^2(1))c_a^2(1) \\ &= (0.887^2)(6.437) + (1 - 0.887^2)(1.0) \\ &= 5.05 + 0.22 \\ &= 5.27 \end{aligned}$$

so  $c_e^2(1)$  makes  $c_a^2(2)$  large

$c_e^2(1)$  is composed of:

$$\begin{aligned} A(1) &= \frac{m_f(1)}{m_f(1) + m_r(1)} \\ &= \frac{48}{48 + 8} = 0.8571 \end{aligned}$$

$$t_e(1) = \frac{t_0(1)}{A(1)} = \frac{19}{0.8571} = 22.17(\text{min})$$

$$\begin{aligned} c_e^2(1) &= c_0^2(1) + \frac{2m_r(1)A(1)(1 - A(1))}{t_0(1)} \\ &= 0.25 + \frac{2(480)(0.8571)(0.1429)}{19} = 6.44 \end{aligned}$$

So **improve breakdown situation at resist apply!** How?

Note that resist apply is problem, though expose is the bottleneck