Analysis of Manufacturing Systems 4AB00

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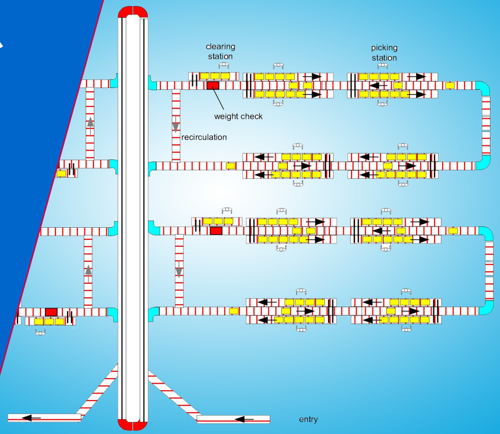

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Contents

Lecture notes on Analysis of Manufacturing Systems

Recap

- Ingredients of a probability model
  - Sample space S, which can be discrete or continuous
  - Events, which are subsets of S
  - Probabilities $P(E)$ of events
- Conditional probability

$$P(E|F) = \frac{P(EF)}{P(F)}, \text{ or } P(EF) = P(E|F)P(F)$$

- Independent events E and F

$$P(EF) = P(E)P(F)$$

- Law conditional probability

$$P(E) = P(E|F_1)P(F_1) + \cdots + P(E|F_n)P(F_n)$$

where events $F_1, \ldots, F_n$ are disjoint and $S = F_1 \cup \cdots \cup F_n$

Recap

- Discrete random variable X takes discrete values, $x_1, x_2, \ldots$
- Function $p_j = P(X = x_j)$ is the probability distribution
- Expected value

$$E(X) = \sum_{j=1}^{\infty} x_j p_j$$

- Variance (measure of variability)

$$\begin{aligned} \mathrm{var}(X) &= E((X - E(X))^2) \\ &= E(X^2) - (E(X))^2. \end{aligned}$$

Recap

- Expectation of sum = sum of expectation

$$E(X+Y) = E(X) + E(Y)$$

- Random variables X and Y are independent if for all x_j, y_i,

$$P(X = x_j, Y = y_i) = P(X = x_j)P(Y = y_i)$$

- Variance of sum = sum of variance (only if X and Y are independent!)

$$\text{var}(X+Y) = \text{var}(X) + \text{var}(Y).$$

Recap

- Bernoulli random variable X with success probability p,

$$P(X = 1) = 1 - P(X = 0) = p.$$

- Binomial random variable X of n independent trials with success prob p,

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots n.$$

- Poisson random variable X with parameter $\lambda > 0$,

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots$$

Property: Poisson + Poisson = Poisson

Recap

- Hypergeometric random variable X with parameters n, R and W,

$$P(X=r) = \frac{\binom{R}{r}\binom{W}{n-r}}{\binom{R+W}{n}}, \quad r = 0, 1, \ldots, n.$$

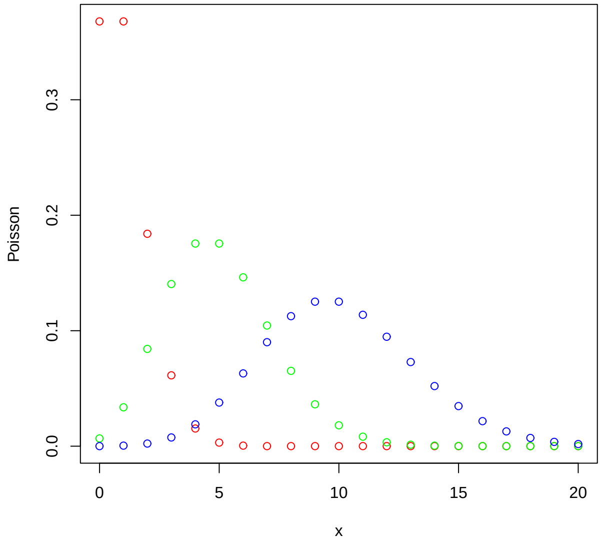
- Geometric random variable X with success probability p,

$$P(X=k) = (1-p)^{k-1}p, \quad k = 1, 2, \ldots$$

- Negative binomial random variable X with parameters r and p,

$$P(X=k) = \binom{k-1}{r-1}(1-p)^{k-r}p^r, \quad k = r, r+1, \ldots$$

Recap



Poisson distributions for $\lambda = 10, 5, 1$. Who is who?

Recap

Example: KIVA system

- N positions (equally spaced) on one side of an aisle in the storage area
- Each position has width of 1 meter
- Each position holds one rack, except for one which is open
- Robot Betty always combines a storage and retrieval action

Questions:

- What is the probability that Betty has to travel up to n meter in the aisle?
- What is the expected distance Betty has to travel in the aisle?

Manufacturing system

- Manufacturing:
  Process of converting raw material into physical products
- System:
  Collection of elements
- Manufacturing system:
  Collection of elements that convert raw material into products

Manufacturing systems at different levels:

- Factory (with areas as elements)
- Area (with cells or groups of machines as elements)
- Cell (with machines as elements)
- Machine (with machine components as elements)

Focus on manufacturing systems at area and cell level

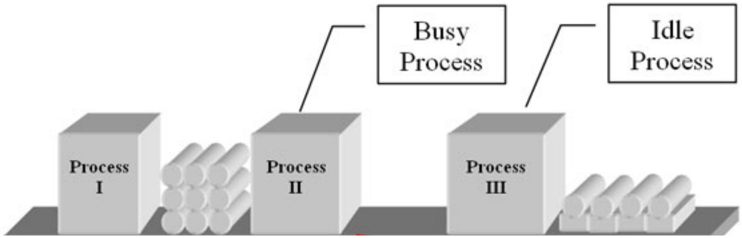
Definitions

- Workstation (or station or workcenter) is collection of machines or workers performing identical functions
- Part is piece of raw material, component, subassembly or assembly worked on at workstation
- Raw material refers to parts purchased outside plant
- Components are pieces assembled into more complex prodcuts
- Subassemblies are assembled products that are further assembled
- Assemblies (or final assemblies) are end items (sold to customers)
- Order (or customer order) is request from customers for particular product, in particular quantity to be delivered on particular (due) date
- Routing is sequence of workstations passed through by part (or job)
- Job (or lot) is set of materials (and information) traversing a routing

Performance measures

- Throughput or throughput rate is number of parts (or jobs) produced per unit time
- Capacity is upper limit on throughput
- Flow time, cycle time, throughput time oor sojourn time is time it takes from release of a job to go through the system and reach the end of the routing
- Work-In-Process (WIP) level is all products from start to end point of a product routing
- Utilization of a workstation is fraction of time it is producing

Buffers



- Manufacturing system transforms material to meet demand
- Buffer is excess resource to correct misalignment between
  - transformation
  - demand
- Three forms of buffers
  - Inventory (extra material in transformation process)
  - Time (delay between demand and satisfaction)
  - Capacity (extra transformation capacity)
- Misalignment due to variability!

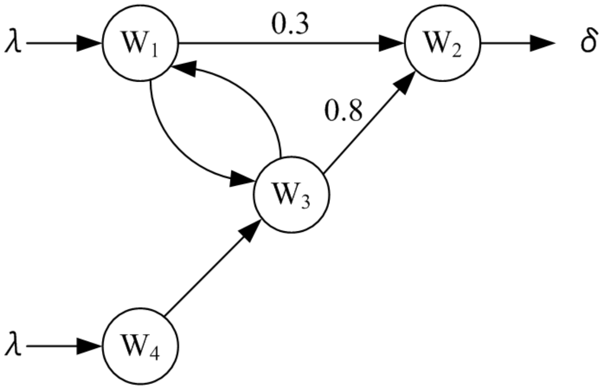
Manufacturing network

- Network of workstations $W_1, \ldots, W_N$
- m_i parallel and identical machines in workstation W_i
- Mean processing time $t_{s,i}$ in workstation W_i
- External inflow λ_i jobs per unit time unit
- Fraction p_{ij} of throughput δ_i of workstation W_i diverted to W_j
- Fraction p_{i0} of throughput is finished and leaves network

Questions:

- What is the capacity of each workstation?
- How many (internal and external) jobs are sent to W_i per unit time?
- Is workstation W_i able to handle this amount of work?

Example



Workstation	m	t_s [hour]
W_1	1	3.0
W_2	1	2.0
W_3	1	1.8
W_4	1	5.4

Conservation of Flow

- Network of workstations $W_1, \ldots, W_N$
- External inflow λ_i jobs per unit time unit
- Fraction p_{ij} of throughput δ_i of workstation W_i diverted to W_j
- Fraction p_{i0} of throughput is finished and leaves network

δ_i is the throughput (or total inflow) of workstation W_i satisfying

$$\delta_i = \lambda_i + \sum_{j=1}^{N} \delta_j p_{ji}, \quad i = 1, \ldots, N.$$

These linear equations have a unique solution, provided the routing is such that every job eventually leaves the manufacturing network

Conservation of Flow

- Network of workstations $W_1, \ldots, W_N$
- m_i parallel and identical machines in workstation W_i
- Mean processing time $t_{s,i}$ in workstation W_i

Capacity of workstation W_i is $m_i/t_{s,i}$, so W_i can handle work if

$$\delta_i < \frac{m_i}{t_{s,i}}$$

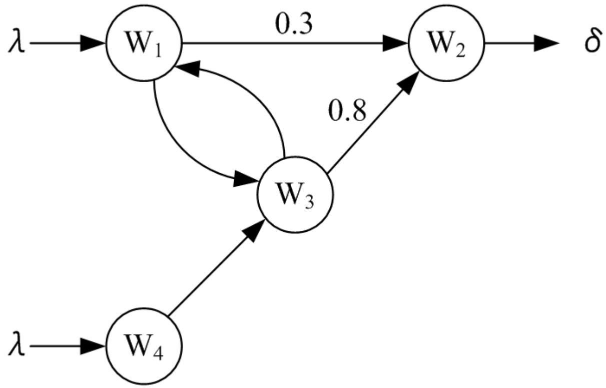
Note: "$=$" is only feasible in deterministic world

Then utilization u_i of machine in workstation W_i is

$$u_i = \frac{\delta_i t_{s,i}}{m_i} < 1$$

Bottleneck workstation W_b is the one with maximal utilization

Example



Workstation	m	t_s [hour]
W_1	1	3.0
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W_3	1	1.8
W_4	1	5.4

Example

- Formulate the flow equations for δ_i.
- Express output rate δ_i of workstation i as function of λ.
- Express utilization u_i of workstation i as function of λ.
- What is the bottleneck workstation?
- What is maximal inflow rate $\lambda_{\max}$?
- What is maximal outflow rate (or maximal throughput) $\delta_{\max}$?

Little's Law

Consider a stable system

- w is the mean WIP level in the system
- δ is the throughput of the system
- φ is the mean flow time in the system

Little's law

$$w = \delta\varphi$$

Remarks:

- Stable means inflow equals outflow (no accumulation of jobs)
- Definition of system is very flexible
  - buffer
  - machine(s)
  - workstation
  - collection of workstations or even whole manufacturing system

Single machine model

Components:

- Generator G sends lots to machine M;
- Machine M processes these lots and sends finished lots to exit E;
- Exit E is doing some book keeping.

Parameters:

- Mean inter-arrival time t_a of the generator G;
- Mean process time t_s of the machine M.

Question: What is the throughput δ?

Note: This model can also be seen as tight two-station line

Object type lot


```
type lot = int;
```

Lots are numbered.

Generator G


```
proc G(chan! lot a; real ta):
    lot x;

    while true:
        a!x;
        delay ta;
        x = x + 1;
    end
end
```

G generates lots with constant inter-arrival times t_a.

Machine M


```
proc M(chan? lot a; chan! lot b; real ts):
    lot x;

    while true:
        a?x;
        delay ts;
        b!x;
    end
end
```

M processes lots with constant process times t_s.

Exit E


```
proc real E(chan? lot a; int n):
    lot x;

    while x < n:
        a?x;
        x = x + 1;
    end;
    exit x / time
end
```

Exit E computes throughput over first n lots.

Single machine model GME


```
model real GME(real ta, ts; int n):
    chan lot a, b;

    run G(a, ta), M(a, b, ts), E(b, n)
end
```

Single machine model

Question: Why is δ for random process times smaller than for constant times?

A is inter-arrival time, S process time and C cycle time,

$$C = \max\{X_a, X_s\}.$$

Then

$$\delta = \frac{1}{E(C)} \leq \frac{1}{\max\{E(A), E(S)\}}$$

with equality only for constant A and S

- Throughput loss is due to starvation ($A > S$) and blocking ($A < S$)
- This misalignment between arrivals and processing is due to variability

Single machine model

Question: How to deal with variations in inter-arrival times and proces times?

Answer: Buffering!

If buffer is sufficiently large, then

$$\delta = \frac{1}{\max\{E(A), E(S)\}}.$$

Buffer machine model $GBME$

```
proc B(chan? lot a; chan! lot b; int N):
    list lot xs;
    lot x;
    while true:
        select
            size(xs) < N, a?x:
                xs = xs + [x]
        alt
            size(xs) > 0, b!xs[0]:
                xs = xs[1:]
        end
    end
end

model real GBME(real ta, ts; int n, N):
    chan lot a, b;
    run G(a, ta), B(a, b, N), M(b, c, ts), E(c, n)
end
```

Question: How to calculate throughput in case of buffers?