Analysis of Manufacturing Systems 4AB00

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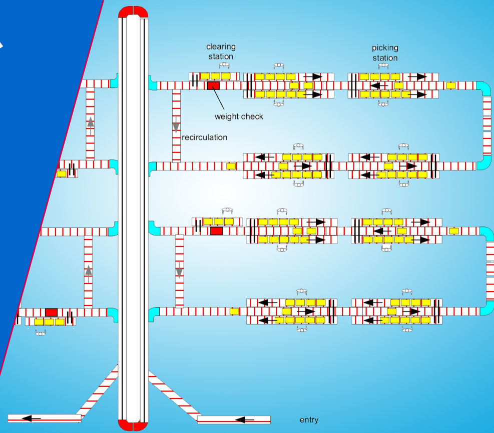

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Lecture notes on Analysis of Manufacturing Systems

- Chapter 3
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Recap: Conditional probability

Conditional probability of X given $Y = y$:

- Discrete random variables X and Y

$$P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

and

$$P(X = x) = \sum_y P(X = x|Y = y)P(Y = y)$$

- Continuous random variables X and Y

$$f_X(x|y) = \frac{f(x, y)}{f_Y(y)}, \quad P(X \leq x|Y = y) = \int_{-\infty}^{x} f_X(u|y)du$$

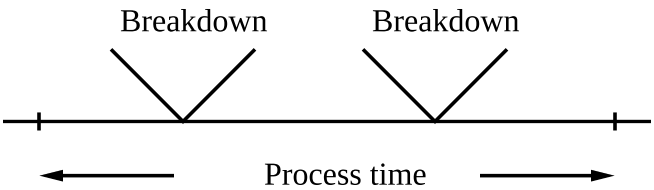
and

$$P(X \leq x) = \int_{-\infty}^{\infty} P(X \leq x|Y = y) f_Y(y)dy$$

Recap: Example

- Process time of item A is exponential with mean 2 hours
- Process time of item B is uniform on $(0, 4)$ hours

Question: What is probability process time of A is less than that of B?



- Machine suffers from breakdowns
- Time between breakdowns is exponential with rate of 1 per hour
- After repair, processing resumes at point where it was interrupted

Question: What is the probability of 2 breakdowns during processing of B?
Question: What is the expected number of breakdowns?

Recap: Conditional expectation

Conditional expectation of X given $Y = y$:

- Discrete random variables X and Y

$$E(X|Y = y) = \sum_x x P(X = x|Y = y),$$

and

$$E(X) = \sum_y E(X|Y = y) P(Y = y).$$

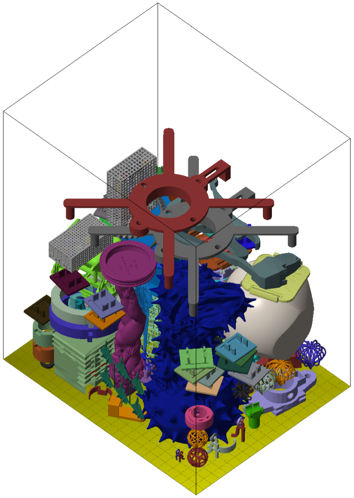
- Continuous random variables X and Y

$$E(X|Y = y) = \int_{-\infty}^{\infty} x f_X(x|y) dx,$$

and

$$E(X) = \int_{-\infty}^{\infty} E(X|Y = y) f_Y(y) dy.$$

Recap: Example



3D printing:

- Expected time to print a tray of height x meter is $18\sqrt{x}$ hours
- Height of tray is uniform between 0.5 and 1 meter

Question: What is the expected time to print a tray?

Variability

- Manufacturing system transforms material to meet demand
- Buffer is excess resource to correct misalignment between
  - transformation
  - demand
- Three forms of buffers
  1. Inventory (extra material in transformation process)
  2. Time (delay between demand and satisfaction)
  3. Capacity (extra transformation capacity)
- Lack of alignment due to variability

Examples of variability

Bad variability:

Cause	Example
Planned outages	setups
	maintenance
Unplanned outages	machine failures
	operator unavailability
Quality problems	rework
	scrap
Operation variation	skill differences
	material variations

Question: Is variability always bad?

Examples of variability

(Potentially) Good variability:

Cause	Example
Product variety	GM
Demand variability	Jiffy Lube (while-you-wait oil change)

Implications of variability

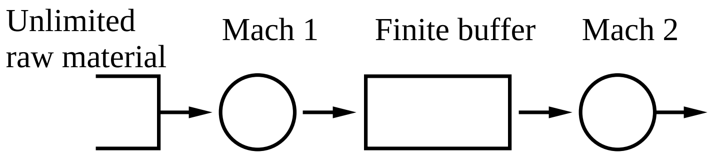
- Increasing variability degrades the performance of a production system
- Variability in a production system will be buffered by some combination of
  1. Inventory
  2. Time
  3. Capacity
- How to buffer for variability depends on the environment!
  Examples:
  – Retailer of ballpoints
  – Emergency service
  – Organ transplants

Pay me now or pay me later

If you do not pay to reduce variability, you will pay in terms of:

- Lost throughput
- Wasted (idle) capacity
- Inflated cycle times
- Larger inventory (WIP) levels
- Long manufacturing lead times
- Poor customer service

Revisit: Buffer machine model



- Process time machine 1 has mean $t_e(1) = 20$ (mins) and cv $c_e(1) = 1$
- Process time machine 2 has mean $t_e(2)$ (mins) and cv $c_e(2)$
- Finite buffer of size N in between two machines
- Machine 1 never starved (always raw material)
- Machine 2 never blocked

Cases:

1. Balanced, moderate cv, large buffer: $t_e(2) = 20$, $c_e(2) = 1$, $N = 10$
2. Balanced, moderate cv, small buffer: $t_e(2) = 20$, $c_e(2) = 1$, $N = 1$
3. Unbalanced, moderate cv, small buffer: $t_e(2) = 10$, $c_e(2) = 1$, $N = 1$
4. Balanced, low cv, small buffer: $t_e(2) = 20$, $c_e(1) = c_e(2) = 0.25$, $N = 1$

Revisit: Buffer machine model

Case	N	$t_e(2)$	$c_e(2)$	δ	$u(2)$	φ	w
1	10	20	1	66	0.92	150	6.9
2	1	20	1	54	0.75	60	2.3
3	1	10	1	67	0.47	36	1.7
4	1	20	0.25	69	0.96	51	2.5

So we pay for:

1. Throughput by long cycle times and high WIP
2. Short cycle times and low WIP by lost throughput
3. Short cycle times and low WIP by wasted capacity
4. High throughput, short cycle times and low WIP by variability reduction

Impact of variability

Variability affects:

- The way material flows through the system
- How much capacity can be utilized

Implications:

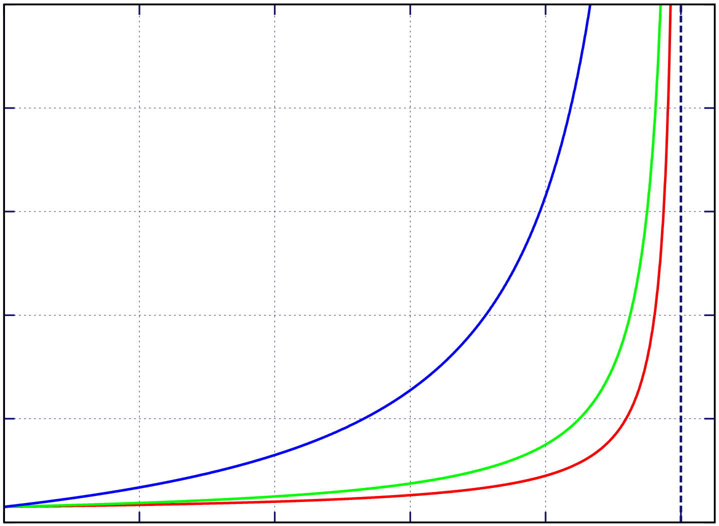
- In a stable system and in the long run, the rate out of a system equals the rate in (minus scrap)
- In steady state, all plants will release work at an average rate which is strictly less than the average capacity
- In a production line, variability early in the line increases cycle time more than equivalent variability later in the line
- If a station increases utilization (without making other changes), average WIP and cycle time will increase in highly nonlinear fashion

Cycle time

Cycle time

$$\varphi \approx \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{u}{1-u}\right) t_e + t_e$$

as a function of the utilization u, $0 < u < 1$:



Variability pooling: Batching

Question:

Which is more variable:
Process time of individual part or that of batch of parts?

- Process time of part with mean t_e and variance σ_e^2

$$c_e(\text{part}) = \frac{\sigma_e}{t_e}$$

- Process time of batch n of parts

$$t_e(\text{batch}) = nt_e,\ \sigma_e^2(\text{batch}) = n\sigma_e^2,$$

$$c_e(\text{batch}) = \frac{\sigma_e(\text{batch})}{t_e(\text{batch})} = \frac{\sqrt{n}\sigma_e}{nt_e} = \frac{c_e(\text{part})}{\sqrt{n}}$$

- Coefficient of variation of batch process time decreases by $1/\sqrt{n}$!

Variability pooling: Batching

Question:

Does this mean that we should produce parts in batches to reduce variability?

Batching

Types of batches:

- Process batch: Many parts processed together
  - Sequential:
    Parts are produced sequentially before change-over to other family
  - Simultaneous:
    Parts are produced simultaneously (e.g., in furnace)
- Transfer batch: Many parts are moved together
  - The smaller the batch the less time waiting to form the batch
  - The smaller the batch the more material handling

Sequential batching interactions

Model:

- Batches of parts arrive at single machine
- Setup between each batch
- Batch size affects
  - Number of setups (and thus utilization)
  - Time spent by part waiting in batch

Question:

What is good choice for batch size?

Buffer Bj2b receives jobs and sends batches of size k:

```
proc Bj2b(chan? job a; chan! list job b; int k):
    list job xs;
    job x;

    while true:
        select
            a?x:
                xs = xs + [x]
        alt
            size(xs) >= k, b!xs[:k]:
            xs = xs[k:]
        end
    end
end
```

Batch processing machine

Machine and exit process:

```
proc M(chan? list job a; chan! list job b; dist real u):
    list job x;

    while true:
        a?x;
        delay sample u;
        b!x;
    end
end

proc real E(chan? list job a; int n):
    int i;
    real sum;
    list job x;

    while i < n:
        a?x;
        for j in range(size(x)):
            sum = sum + (time - x[j]);
        end;
        i = i + size(x);
    end;
    exit sum / n
end
```

Batch processing machine

Buffer batch machine model:

```
model real GIBj2bME(real ta, te; int k; int n):
    chan lot a;
    chan list job b, c;

    run G(a, exponential(ta)),
        Bj2b(a, b, k),
        M(b, c, constant(te)),
        E(c, n)
end
```

Sequential batching interactions

- k is batch size
- r_a is arrival rate of parts (parts per hour)
- c_a is coefficient of variation of inter-arrival time of parts
- t_s is time to perform setup (hour)
- σ_s is standard deviation of setup
- t_p is mean time to process single part
- σ_p is standard deviation of process time of single part

Sequential batching interactions

- Batches arrive at rate r_a/k
- Coefficient of variation of inter-arrival time of batches

$$c_a(\text{batch}) = \frac{c_a}{\sqrt{k}}$$

- Effective process of batch

$$t_e = t_s + kt_p, \quad c_e = \frac{\sqrt{\sigma_s^2 + k\sigma_p^2}}{t_s + kt_p}$$

- $u = \frac{r_a}{k} t_e = r_a \left(\frac{s}{k} + t_p\right)$
- Stability: $u < 1$, so

$$k > \frac{r_a t_s}{1 - r_a t_p}$$

Sequential batching interactions

- Mean flow time (of batch)

$$\varphi = \left(\frac{c_a^2(\text{batch}) + c_e^2}{2}\right)\left(\frac{u}{1-u}\right) t_e + t_e$$

Question: How to include waiting of part to form batch?

- Mean wait-to-batch time

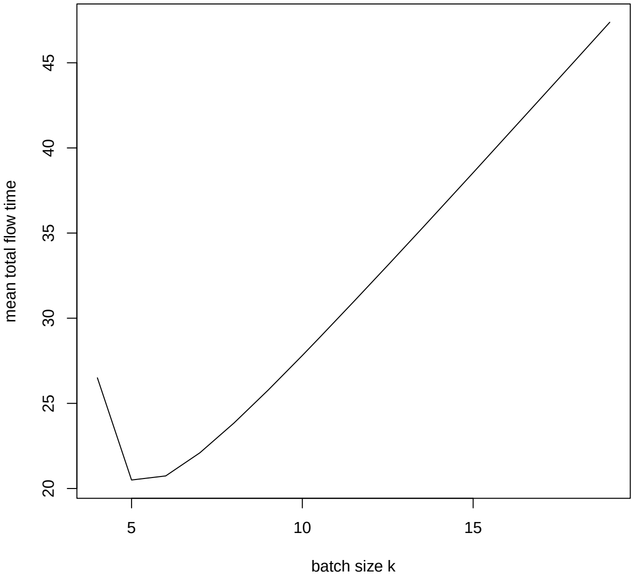
$$\frac{k-1}{2}\frac{1}{r_a}$$

- Mean total flow time (of part)

$$\varphi = \frac{k-1}{2r_a} + \left(\frac{c_a^2(\text{batch}) + c_e^2}{2}\right)\left(\frac{u}{1-u}\right) t_e + t_e$$

Sequential batching interactions

Example: $r_a = 0.4, c_a = 1, t_s = 5, \sigma_s = 2.5, t_p = 1, \sigma_p = 0.5$



Sequential batching interactions

Job splitting: Individual parts are sent downstream as soon as processed

Question:

What is mean flow time in case of job splitting?

- Mean effective process time of part

$$t_s + \frac{k+1}{2} t_p$$

- Mean total flow time

$$\varphi = \frac{k-1}{2r_a} + \left(\frac{c_a^2(\text{batch}) + c_e^2}{2}\right)\left(\frac{u}{1-u}\right) t_e + t_s + \frac{k+1}{2} t_p$$

Sequential batching interactions

Observations:

- Minimum process batch to yield stable system may be greater than 1
- For large batch sizes, flow time grows proportionally with batch size
- Batch size minimizing flow time may be greater than 1