Assignment 3

We consider a production system which combines make-to-order and make-to-stock (see figure 1). There are two types of products, standard and non-standard products. When there are no orders, the production resource (e.g., machine or worker) is used to produce standard products until the stock reaches a a certain maximum level, M say. A customer asking for a standard product receives it directly (if possible) from stock. If the stock is empty the customer order joins the queue. Non-standard, customer specific products are never delivered from stock, but always produced to order. Orders for standard products arrive according to a Poisson process with rate λ_1, the arrival of customer specific orders is Poisson as well with rate λ_2. Orders require one product at a time. The production times for both types of products are exponential with the same mean $1/\mu$. Production to stock is preempted by production to order.

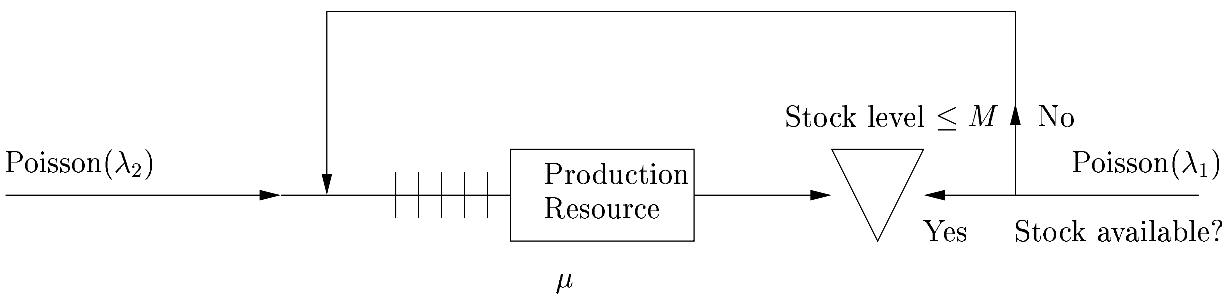

Figure 1: Production sytem combining make-to-order and make-to-stock

a. For which values of the parameters λ_1, λ_2, μ and M is the system stable?

b. Describe this system as a continuous time Markov process, and determine, for various values of the parameters, the equilibrium distribution as well as performance characteristics such as,

   - the mean production lead time of standard and non-standard orders;
   - the mean stock level;
   - fraction of standard orders directly satisfied from stock.

   In particular, show how these performance characteristics depend on the maximal stock level M.

c. Also compute, for various values of the parameters, the minimal value of M required to achieve 80% (resp. 90%) of the reduction of the mean production lead time of standard and non-standard orders, respectively, that is possible by production to stock.