A CLASS OF MARKOV PROCESSES

ON

SEMI-INFINITE STRIP

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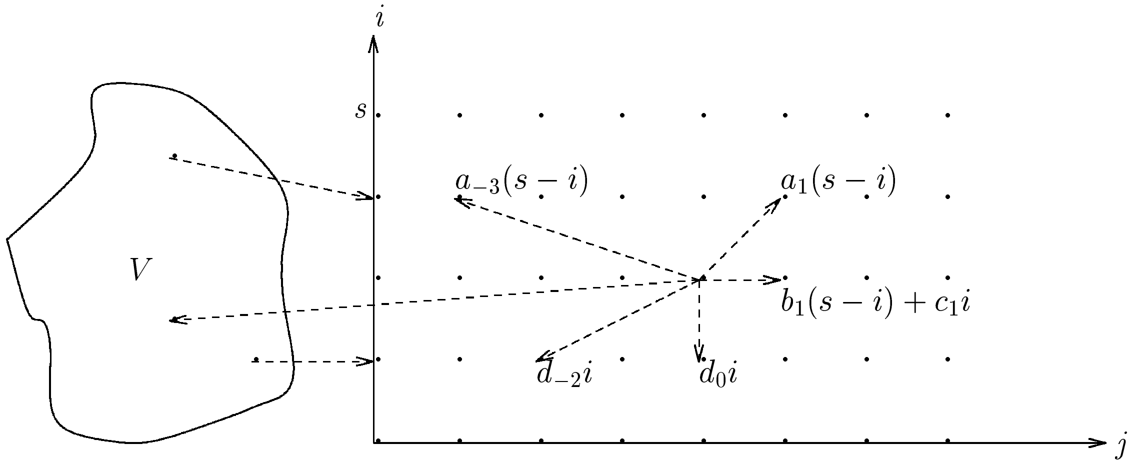
Model



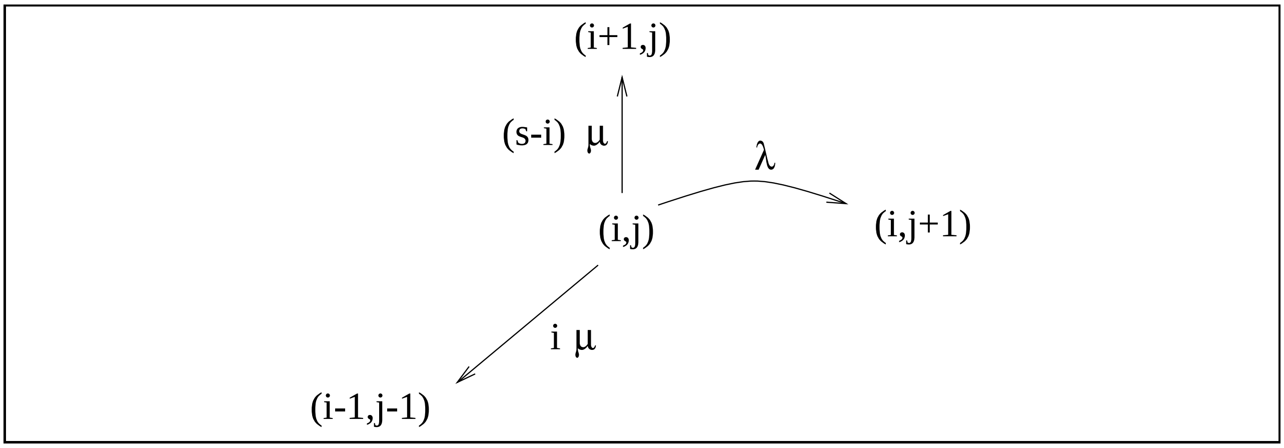
Transitions ($k=-j+1,-j+2,\ldots,0,1$):

- From (i,j) to $(i+1,j+k)$: rate $a_k(s-i)$.
- From (i,j) to $(i,j+k)$: rate $b_k(s-i)+c_k i$.
- From (i,j) to $(i-1,j+k)$: rate $d_k i$.
- From (i,j) to V: total rate $\sum_{k=-\infty}^{-j}[a_k(s-i)+b_k(s-i)+c_k i+d_k i]$.

EXAMPLES:

1. Train model

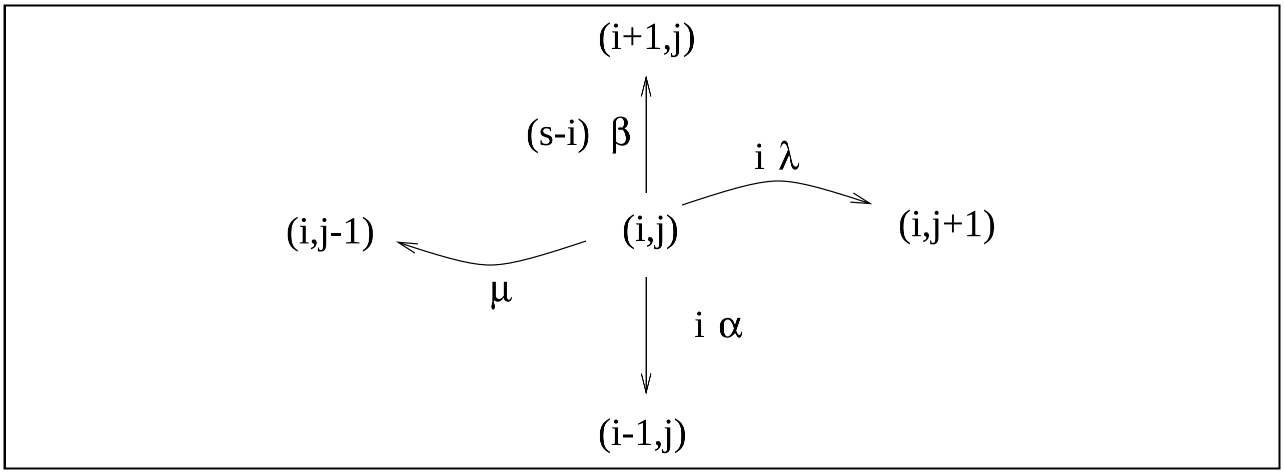
2. $M/E_2/s$
   i = number of servers working on phase 2
   j = number of customers in the queue



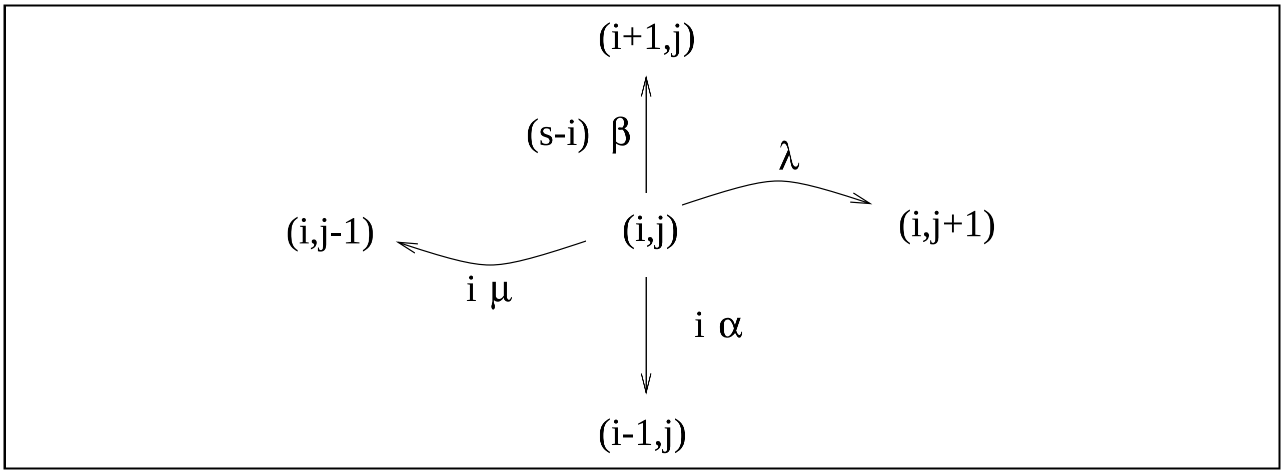
3. $M/H_2/s$

4. $M/C_2/s$

4. $\sum IPP/M/1$
i = number of Poisson sources in on-state
j = number of customers in the system



5. $M/M/s$ with service interruptions
i = number of servers that is up



Approach to find probabilities $p(i,j)$:

1. First construct a *basis of solutions* of the form
$$p(i,j) = y(i)x^j$$
satisfying the equilibrium equations in the strip

2. Use the solutions in this set to construct a linear combination which also satifies the boundary equations

Substitution of

$$p(i,j) = y(i)x^j$$

into equilibrium equations in the strip gives

$$\begin{aligned}
&((s-i)(A(1)+B(1))+i(C(1)+D(1)))xy(i)\\
&= (i+1)D(x)y(i+1)\\
&\quad +((s-i)B(x)+iC(x))y(i)\\
&\quad +(s-i+1)A(x)y(i-1),
\end{aligned}$$

where

$$A(x) := \sum_{k=-\infty}^{1} a_k x^{1-k},$$

and $B(x)$, $C(x)$ and $D(x)$ are similar.

Use $Y(z) = \sum_{i=0}^{s} y(i) z^i$

$$\frac{Y'(z)}{Y(z)} = \frac{E(x)}{z - z_1(x)} + \frac{s - E(x)}{z - z_2(x)}$$

so $Y(z) = K(z - z_1(x))^{E(x)} (z - z_2(x))^{s - E(x)}$

Find x_k such that $E(x_k) = k$ for $k = 0, \ldots, s$. This can be rewritten as:

$$2k - s = G(x_k)$$

for some function $G(x)$.

Lemma: For all $m = 0, \ldots, s$ the equation

$$2k - s = G(x)$$

has a unique solution $x = x_k$ in $(-1, 1)$.

Theorem: $p(i, j)$ on strip can be expressed as:

$$p(i, j) = \sum_{m=0}^{s} C_m y_m(i) x_m^j$$

where C_k follow from boundary equations.