A MULTI-SERVER QUEUEING MODEL FOR TRAIN MAINTENANCE

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OVERVIEW:

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2. Application
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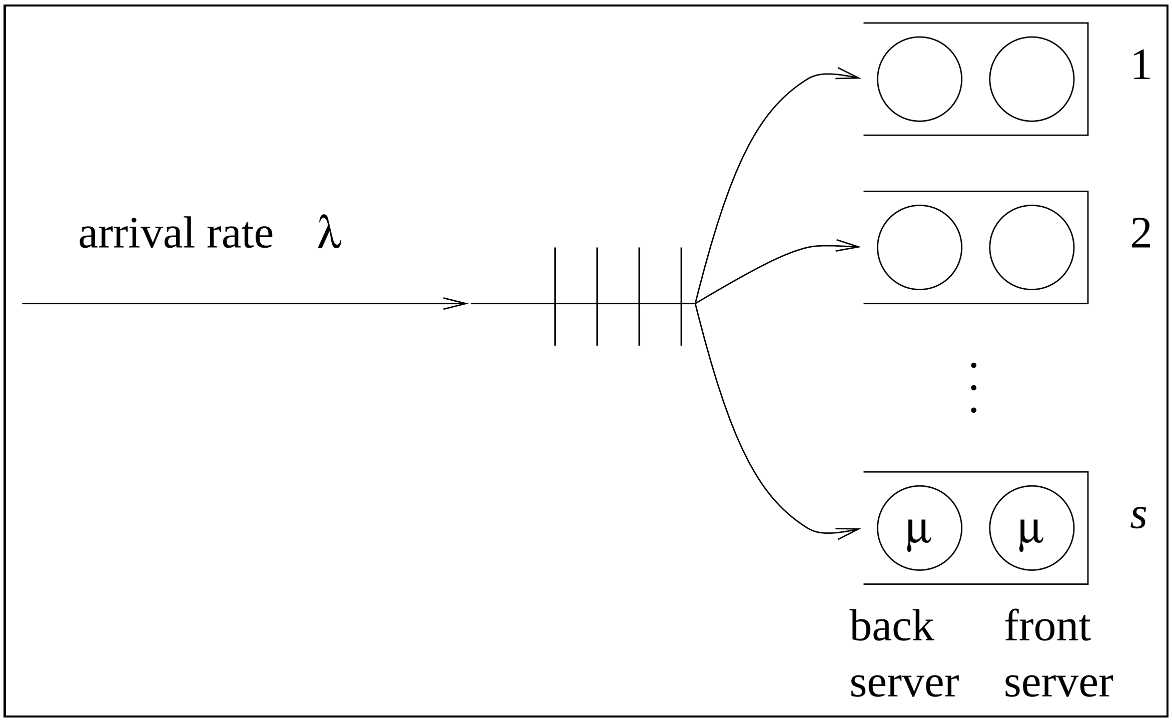
1. Model description



Poisson arrivals (λ)

Exponential service times (μ)

Groups of 2 servers

<u>Locking</u> of front customer

2. Application

Maintenance of Dutch Railway

7 “terminal” tracks

Each track may hold 6 train-units

Trains consist of 2,3 or 4 train-units

3. Capacity analysis

Heavy traffic:

$\alpha :=$ fraction time front server works $= \frac{1}{2}$

so stability condition $\lambda < (1 + \alpha)\mu s$

General service times:

For example,

Erlang: $\frac{1}{2} < \alpha < \frac{2}{3}$

Hyperexponential: $\frac{1}{4} < \alpha < \frac{1}{2}$

4. Queue length analysis

Continuous time Markov chain

Unsaturated states: (x_0, x_1, x_2, x_3)

- x_0: both servers empty

- x_1: back server empty

- x_2: front server blocked

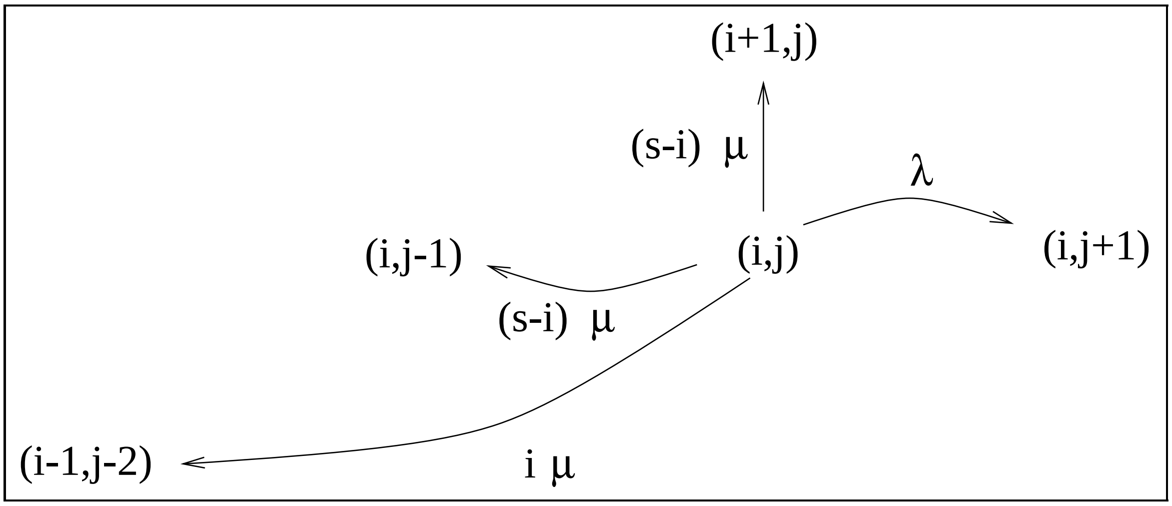
- x_3: both servers busy

Saturated states: (i,j)

- i: number of locked customers

- j: number of customers waiting in queue

Flow diagram:



Approach:

1. First construct solutions of the form
$$p(i,j) = y(i)x^j$$
satisfying the equilibrium equations in the saturated states

2. Use the solutions in this set to construct a linear combination which also satifies the boundary equations

1. Equilibrium equations saturated states

$$\begin{aligned} p(i,j)(\lambda+(2s-i)\mu) &= p(i,j-1)\lambda \\ &+ p(i,j+1)(s-i)\mu \\ &+ p(i-1,j)(s-(i-1)\mu) \\ &+ p(i+1,j+2)(i+1)\mu \end{aligned}$$

2. Assume $p(i,j) = y(i)x^j$ which yields

$$y(i)[-\lambda+(\lambda+(2s-i)\mu)x-(s-i)\mu x^2]$$
$$= y(i-1)x(s-(i-1)\mu) + y(i+1)x^3(i+1)\mu$$

for $i = 0, 1, \cdots, s$

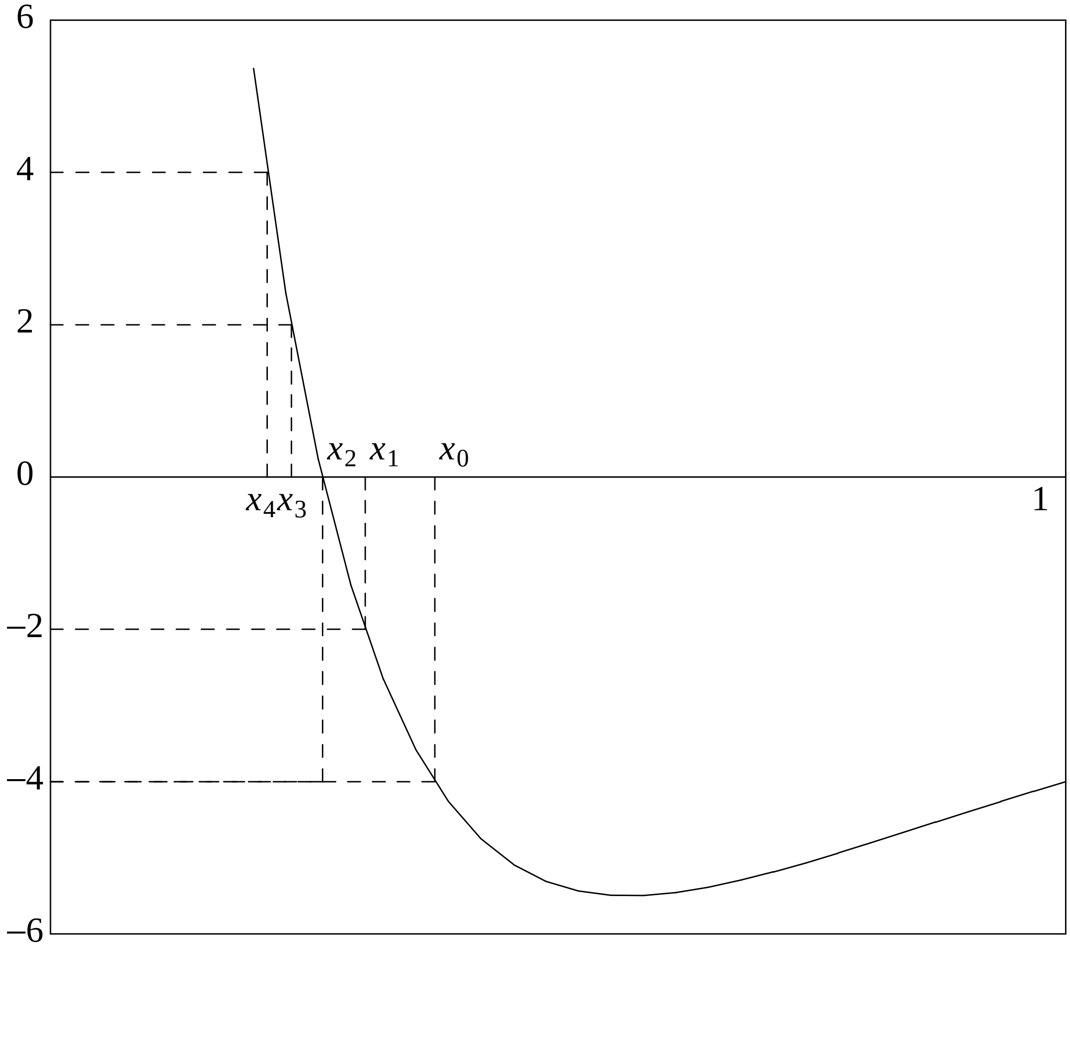
3. Use $Y(z) = \sum_{i=0}^{s} y(i)z^i$

$$\frac{Y'(z)}{Y(z)} = \frac{E(x)}{z-z_1(x)} + \frac{s-E(x)}{z-z_2(x)}$$

so $Y(z) = K(z-z_1(x))^{E(x)}(z-z_2(x))^{s-E(x)}$

4. Find x_k such that $E(x_k) = k$ for $k = 0, \ldots, s$. This can be rewritten as:

$$2k - s = \frac{s(x_k - 3) + 2\lambda(1 - x_k)/(\mu x_k)}{\sqrt{5x_k^2 - 2x_k + 1}}$$



5. Solution for saturated states

$$p(i,j) = \sum_{k=0}^{s} C_k y_k(i) x_k^j$$

where C_k follow from boundary equations

5. Waiting time analysis

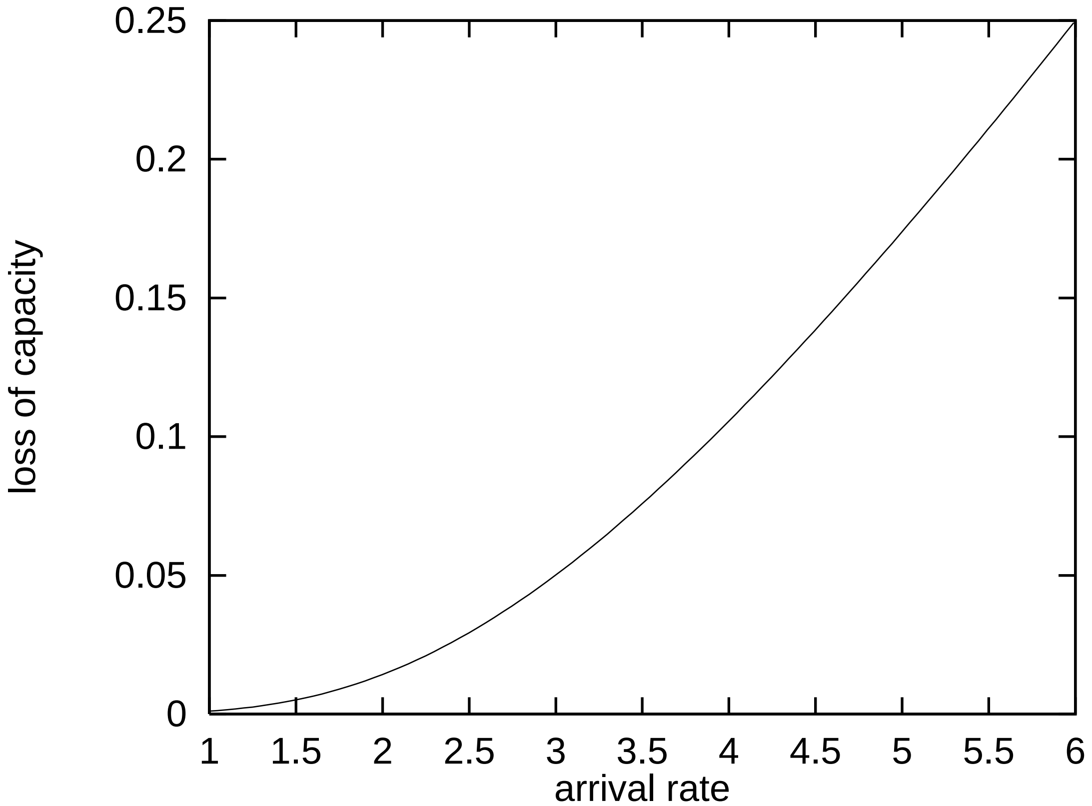
Waiting time is finite mixture of exponentials

$$\Pr[W > t] = \sum_{k=0}^{s} C_k \left(\sum_{i=0}^{s} y_k(i) \right) \frac{1}{1 - x_k} e^{\lambda(1-1/x_k)t}$$

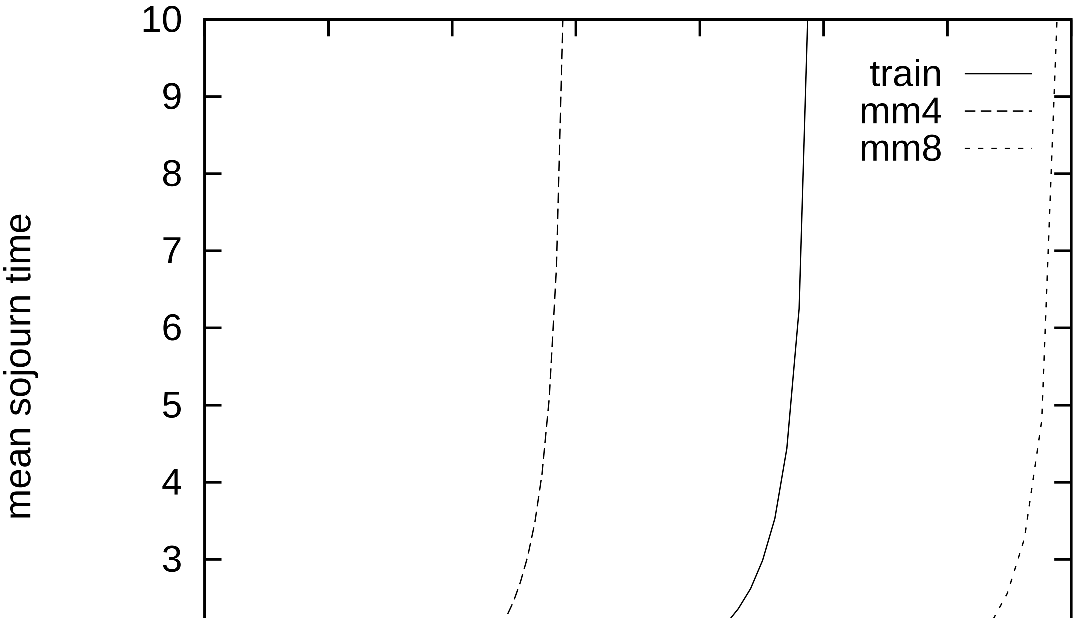
6. Numerical results

Basic example: $s = 4$, $\mu = 1$

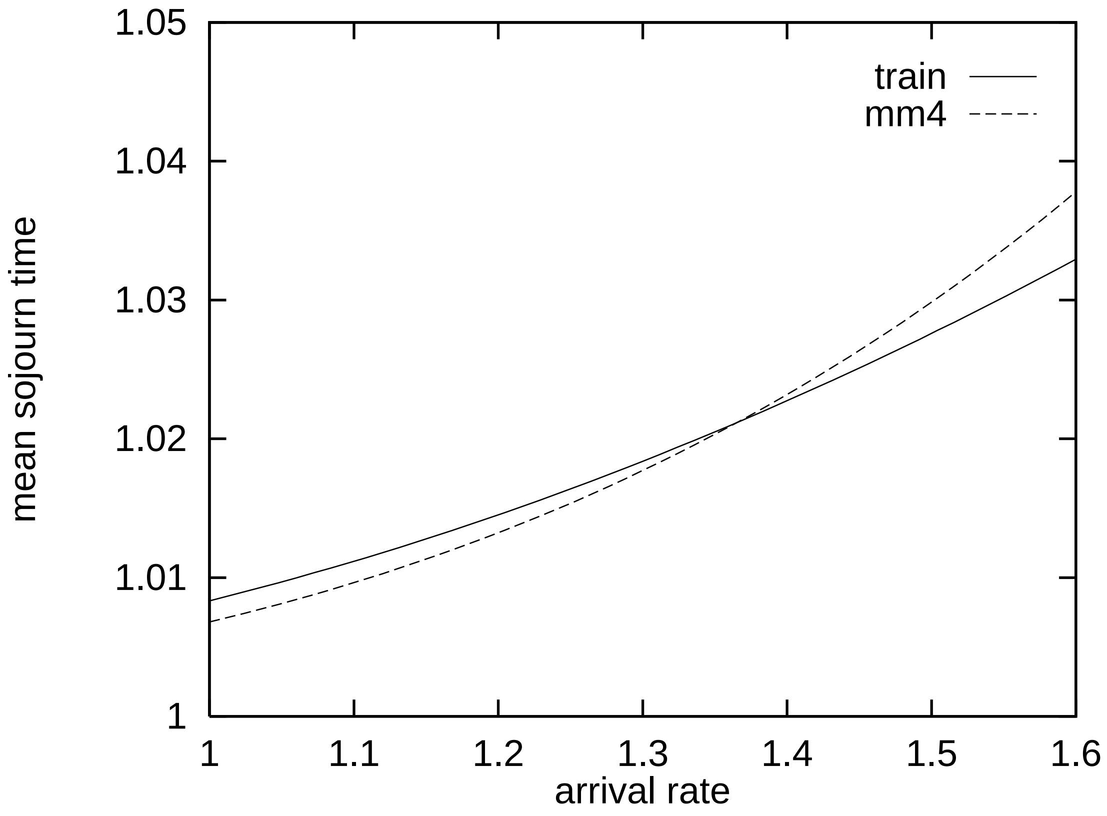
Loss of service capacity



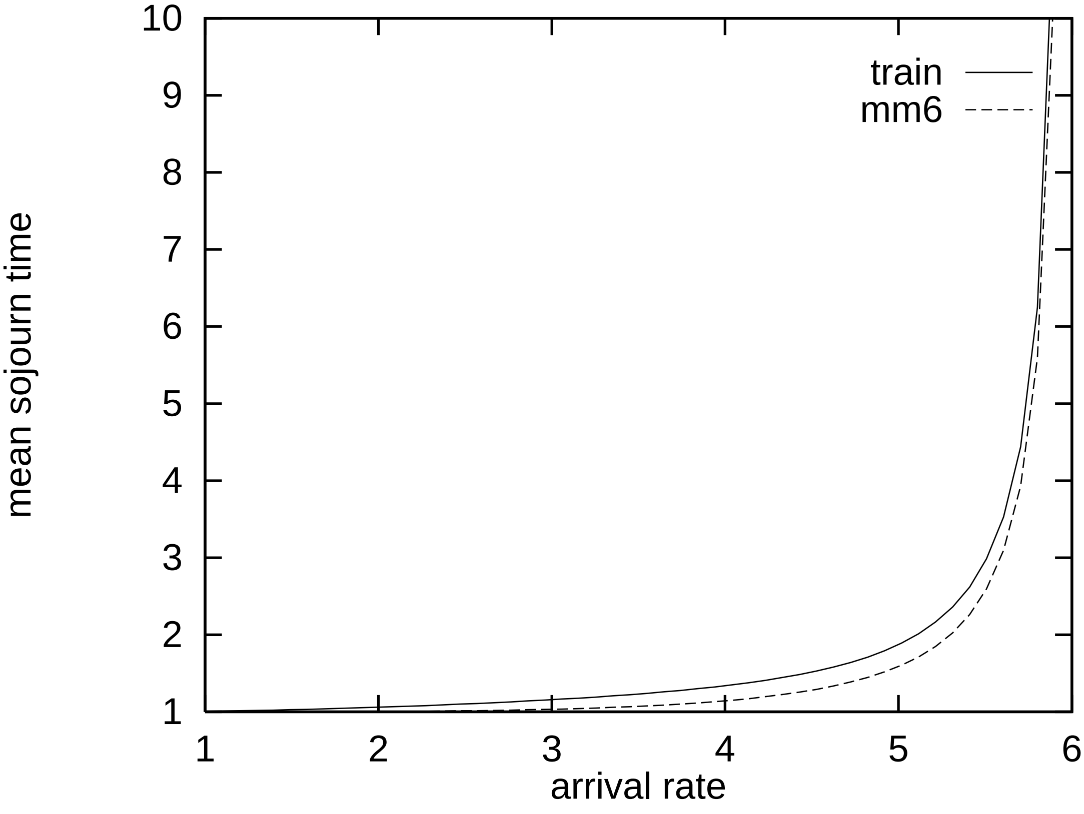
Trainmodel (4 tracks) versus $M/M/4$ and $M/M/8$



Trainmodel (4 tracks) versus $M/M/4$ in light traffic



Train model (4 tracks) versus $M/M/6$



7. Conclusions

exponential multi-server model

effect of locking

loss of maintenance capacity: upto 25%

light traffic: use only half of the track capacity

train model worse than $M/M/\frac{3}{2}s$ model