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Facility Logistics Management

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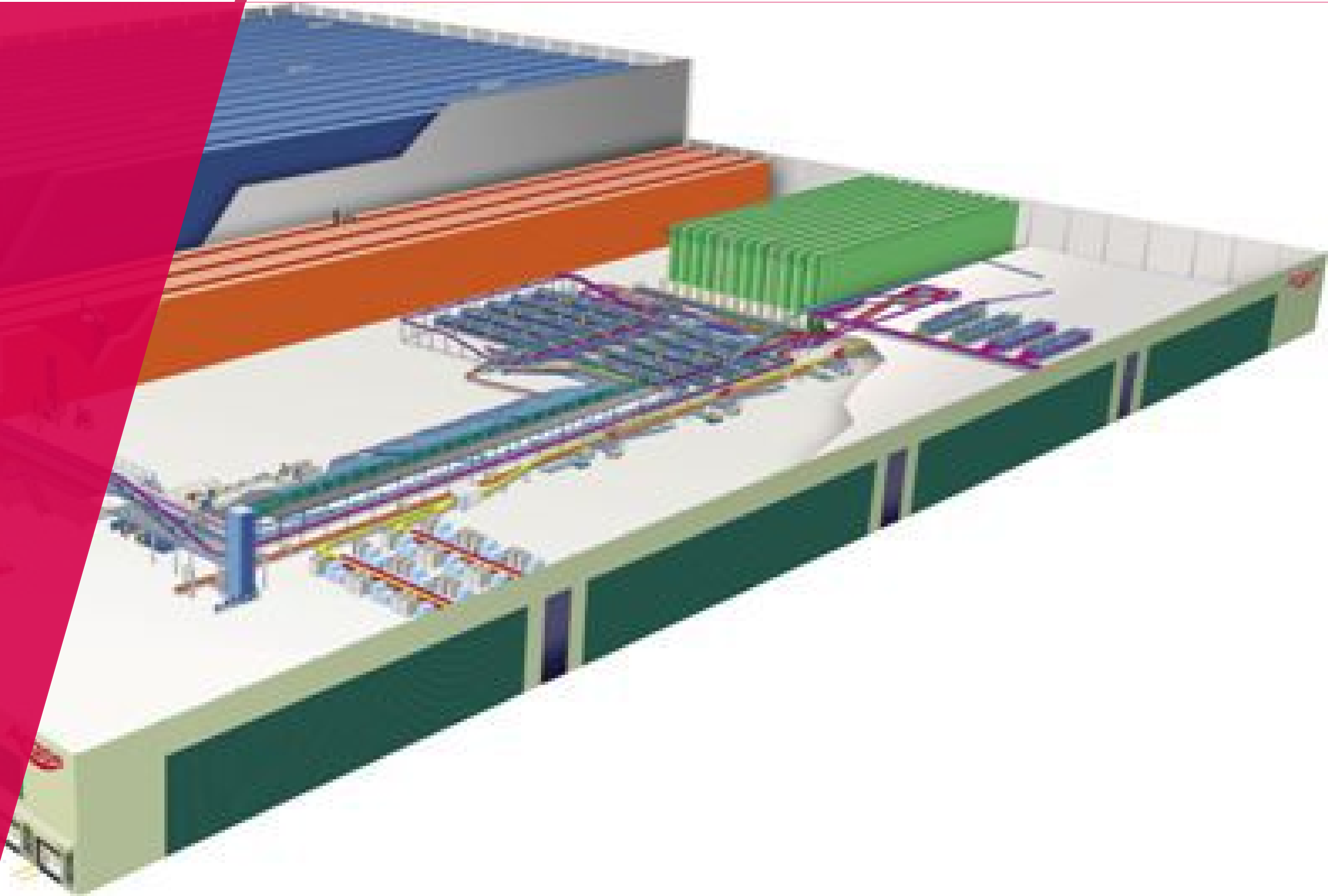
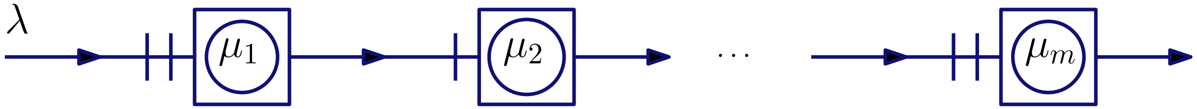


Where innovation starts

Exponential flow lines



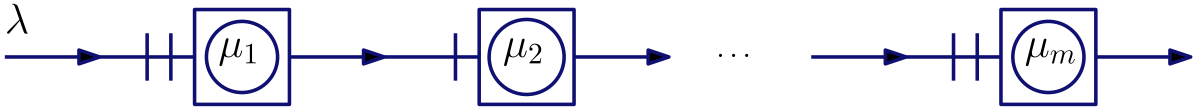
- Jobs arrive according to Poisson process with rate λ
- Service times in single-server station i are exponential with rate μ_i
- Stability: $\rho_i = \frac{\lambda}{\mu_i} < 1$ for all i
- Question: What are mean number and mean flow time in each station?
- Answer: Output of $M/M/1$ is Poisson, so every station is $M/M/1$

$$E(L_i) = \frac{\rho_i}{1-\rho_i}, \quad E(S_i) = \frac{E(L_i)}{\lambda} = \frac{1}{1-\rho_i}\frac{1}{\mu_i}, \quad i = 1, \ldots, m$$

- Mean total flow time

$$E(S) = \frac{E(L)}{\lambda} = \frac{\sum_{i=1}^{m} E(L_i)}{\lambda}$$

Exponential flow lines: Workload allocation



- Jobs arrive according to Poisson process with rate λ
- Service times at server i are exponential with mean $w_i = \frac{1}{\mu_i}$
- Mean total work content of jobs is W
- Question: How to allocate w_i so as to minimize $\mathrm{E}(L)$?
- Answer: Solution to

$$\min \mathrm{E}(L) = \sum_{i=1}^{m} \frac{\lambda w_i}{1 - \lambda w_i}$$

subject to

$$\sum_{i=1}^{m} w_i = W, \quad 0 \leq \lambda w_i < 1, \quad i = 1, 2, \ldots, m$$

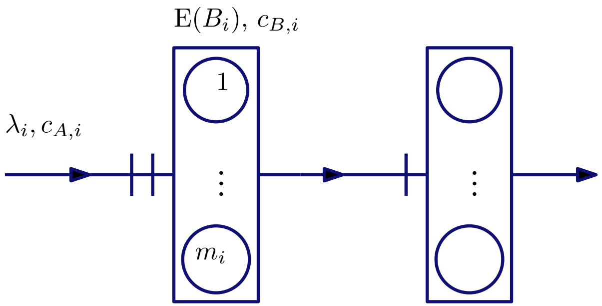
is $w_i = \frac{W}{m}$ for all i, so balance the line!

Exponential flow lines: Impact of unbalance

- Flow line with 4 servers, labeled 1, 2, 3, 4
- Arrival rate $\lambda = 1$
- Mean service time at station i is w_i
- Average work load per station $\rho = \frac{1}{4}(\rho_1 + \rho_2 + \rho_3 + \rho_4)$ where $\rho_i = \lambda w_i$
- Mean number in system

ρ	w_1	w_2	w_3	w_4	$E(L_1)$	$E(L_2)$	$E(L_3)$	$E(L_4)$	$E(L)$
0.80	0.80	0.80	0.80	0.80	4.0	4.0	4.0	4.0	16.0
0.80	0.85	0.65	0.90	0.80	5.7	1.9	9.0	4.0	20.5
0.95	0.95	0.95	0.95	0.95	19.0	19.0	19.0	19.0	76.0
0.95	0.96	0.93	0.97	0.94	24.0	13.3	32.3	15.7	85.3

General flow lines



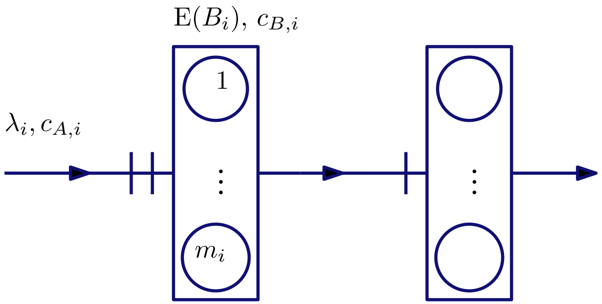
- Stable:

$$\rho_i = \frac{\lambda_i E(B_i)}{m_i} < 1$$

- Variability interactions: Waiting time approximation in station i

$$E(W_i) \approx \frac{1}{2}(c_{A,i}^2 + c_{B,i}^2)\,\frac{\Pi_{W,i}}{1-\rho_i}\,\frac{E(B_i)}{m_i}$$

General flow lines



- **Conservation of flow:** Flow into station $i+1$ is flow out of station i is flow into station i

$$\lambda_{i+1} = \lambda_i$$

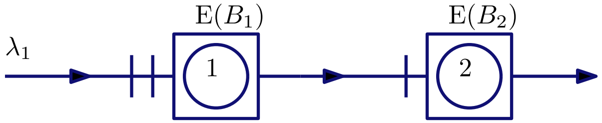
- **Propagation of variability:**

$$c_{D,i}^2 = 1 + (1-\rho_i^2)(c_{A,i}^2 - 1) + \frac{\rho_i^2}{\sqrt{m_i}}(c_{B,i}^2 - 1)$$

- **Linking of workstations:** Departures from workstation i are arrivals to workstation $i+1$

$$c_{A,i+1} = c_{D,i}$$

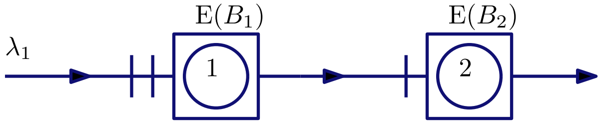
Serial flow lines: Example



- Machine 1 has Poisson inflow with rate $\lambda_1 = 2$
- Process times on Machine 1 are Constant with $E(B_1) = \frac{1}{3}$ and on Machine 2 are Uniform$(0, \frac{4}{5})$ with $E(B_2) = \frac{2}{5}$
- Question: What is mean total flow time of jobs?
- Answer: $\rho_1 = \lambda_1 E(B_1) = \frac{2}{3}$, $\frac{1}{2}(c_{A,1}^2 + c_{B,1}^2) = \frac{1}{2}(1+0) = \frac{1}{2}$

$$E(S_1) \approx \frac{1}{2}(c_{A,1}^2 + c_{B,1}^2)\frac{\rho_1}{1-\rho_1} E(B_1) + E(B_1) = \frac{2}{3}$$

Serial flow lines: Example

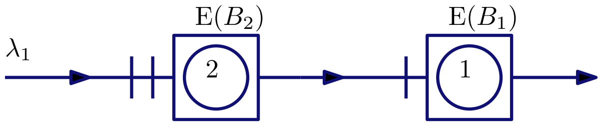


- Machine 1 has Poisson inflow with rate $\lambda_1 = 2$
- Process times on Machine 1 are Constant with $\mathrm{E}(B_1) = \frac{1}{3}$ and on Machine 2 are Uniform$(0, \frac{4}{5})$ with $\mathrm{E}(B_2) = \frac{2}{5}$
- Question: What is mean total flow time of jobs?
- Answer: $\lambda_2 = \lambda_1 = 2$ and $c_{A,2}^2 = c_{D,1}^2 = \rho_1^2 c_{B,1}^2 + (1 - \rho_1^2) c_{A,1}^2 = \frac{5}{9}$
  $c_{B,2}^2 = \frac{1}{3}$, so $\frac{1}{2}(c_{A,2}^2 + c_{B,2}^2) = \frac{1}{2}(\frac{5}{9} + \frac{1}{3}) = \frac{4}{9}$ and $\rho_2 = \lambda_2 \mathrm{E}(B_2) = \frac{4}{5}$

$$\mathrm{E}(S_2) \approx \frac{1}{2}(c_{A,2}^2 + c_{B,2}^2)\,\frac{\rho_2}{1-\rho_2}\,\mathrm{E}(B_2) + \mathrm{E}(B_2) = 1\frac{1}{9}$$

So total flow time is $\mathrm{E}(S) = \mathrm{E}(S_1) + \mathrm{E}(S_2) \approx 1\frac{7}{9} = 1.77$ (Simulation: $\mathrm{E}(S) = 1.86$)

Serial flow lines: Example



- Reverse machines: First Machine 2 and then Machine 1
- Question: Does mean total flow time increase or decrease?
- Answer: Flow time for reversed system

$$E(S) \approx 1.99$$

  which is greater than for original configuration
- Question: Why?
- Answer: Variability propagates!
  Process variability on Machine 2 is higher than on Machine 1, so it is better to locate it toward end of line
- Lesson: Variability early in line is more disruptive than variability late in line!