1. Consider a branching process in discrete time with $X_0 = 1$. Compute the extinction probability π_0 in the following two cases for the offspring distribution:

a. $P(Z = 0) = 1/5, P(Z = 1) = 3/5$, and $P(Z = 2) = 1/5$.

b. $P(Z = 0) = 1/5, P(Z = 1) = 2/5$, and $P(Z = 2) = 2/5$.

c. Solve part a and b, but now for $X_0 = 2$.

2. Let $X_0 = 1$, define $P_j = P(Z = j)$ and let $P_1 = \alpha$, $P_0 = P_2 = P_3 = \frac{1-\alpha}{3}$, with $\alpha \in (0, 1)$.

a. Determine $E[X_n]$.

b. Determine the extinction probability π_0.

c. Answer part a and b again, but now when X_0 is random with $P(X_0 = n) = (1/2)^n$, $n \geq 1$.

3. Consider the following continuous-time analogue of a Branching process: Each particle has a lifetime which is exponentially distributed with rate μ. During its lifetime, the particle generates children with rate λ. Let X_t be the number of particles at time t.

a. Assume $X_0 = 1$. Let π_0 be the extinction probability. Derive an equation for π_0. Consider both the case $\lambda \leq \mu$ and $\lambda > \mu$.

b. Show that $X_t, t \geq 0$, is a birth and death process, and provide the birth and death rates.

4. Exercise 7.2.

5. Exercise 7.4.