1.

a. We have

$$v_0 = \lambda, \quad v_1 = \lambda + \mu, \quad v_2 = \mu,$$

and

$$P_{01} = P_{20} = 1, \quad P_{10} = 1 - P_{12} = \frac{\mu}{\lambda + \mu}.$$

b. Let $T_i (i = 1, 2)$ denote the time to go from state i to 0. Then $E[T_2] = 1/\mu$ and

$$E[T_1] = \frac{1}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} \cdot E[T_2] = \frac{1}{\mu}.$$

2. Let us assume that the state is (n, m). Male i mates at a rate λ with female j, and therefore it mates at a rate λm. Since there are n males, mating occurs at a rate λnm. Therefore

$$v_{n,m} = \lambda nm.$$

Since any mating is equally likely to result in a female as in a male, we have

$$P_{(n,m);(n+1,m)} = P_{(n,m);(n+1,m)} = \frac{1}{2}.$$

3. This is not a birth and death process since we need more information than just the number working. We must also know which machine is working. We can analyze it by letting the states be

b	:	both machines are working
1	:	1 is working, 2 is down
2	:	2 is working, 1 is down
$d1$	:	both are down, 1 is being repaired
$d2$	:	both are down, 2 is being repaired.

Then

$$v_b = \mu_1 + \mu_2, \quad v_1 = \mu_1 + \mu, \quad v_2 = \mu_2 + \mu, \quad v_{d1} = v_{d2} = \mu,$$

and

$$\begin{aligned} P_{b,1} = 1 - P_{b,2} &= \frac{\mu_2}{\mu_1 + \mu_2}, \\ P_{1,b} = 1 - P_{1,d2} &= \frac{\mu}{\mu_1 + \mu}, \\ P_{2,b} = 1 - P_{2,d1} &= \frac{\mu}{\mu + \mu_2}, \\ P_{d1,1} = P_{d2,2} &= 1. \end{aligned}$$

4.

a. Yes.

b. It is a pure birth process.

c. If there are i infected individuals then since a contact will involve an infected and an uninfected individual with probability $i(n-i)/\binom{n}{2}$, it follows that the birth rates are $\lambda_i = \lambda i(n-i)/\binom{n}{2}$, $i = 1, \ldots, n$. Hence,

$$E[\text{time all infected}] = \frac{n(n-1)}{2\lambda} \sum_{i=1}^{n} \frac{1}{i(n-i)}.$$

5. Starting with $E[T_0] = \frac{1}{\lambda_0} = \frac{1}{\lambda}$, employ the identity

$$E[T_i] = \frac{1}{\lambda_i} + \frac{\mu_i}{\lambda_i} E[T_{i-1}]$$

to succesively compute $E[T_i]$ for $i = 1, 2, 3, 4$.

a. $E[T_0] + \cdots + E[T_3]$.

b. $E[T_2] + E[T_3] + E[T_4]$.