Exam Queueing Theory
Monday, June 25, 2012, 14.00 – 17.00.

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1. Passengers are brought with small vans from the airport to hotels nearby. At one of those hotels on average 6 vans per hour arrive according to a Poisson process. With probability 1/4 a van brings 2 guests for the hotel, with probability 1/4 only one guest and with probability 1/2 no guests at all. At the reception of the hotel there is always one receptionist present. It takes an exponential time with a mean of 5 minutes to check in a guest.

   a) Determine the distribution of the number of guests at the reception.

   b) What is the mean waiting time of an arbitrary guest at the reception?

   c) What is the mean waiting time of a guest who arrived in a group of two and is served after the other guest arriving at the same time?

2. Two different product types, product type A and product type B, are produced to order on a single machine. The orders for the two product types arrive according to independent Poisson processes. The production time of a type A product is exponentially distributed with a mean of 8 minutes. The production time of a type B product is exponentially distributed with a mean of 4 minutes. The rate at which orders for type A products arrive is 3 orders per hour. The rate at which orders for type B products arrive is 5 orders per hour. The orders are served in order of arrival, independent of the product type.

   a) Determine the expected waiting time of orders for type A products and of orders for type B products. Also determine the expected number of orders for type A products and orders for type B products in the system.

   b) What is the expected amount of work in the system at an arbitrary instant (i.e., the expected time needed to serve all the orders present in the system)?

   One decides to give orders for type B products priority over order for type A products. However, when the machine is producing a type A product, this production can not be interrupted.

   c) Again determine the expected waiting time of orders for type A products and of orders for type B products. Also determine the expected number of orders for type A products and orders for type B products in the system in this situation.

   d) What is in this situation the expected amount of work in the system at an arbitrary instant? Compare the answer with the expected amount of work in the system in part b and explain the result.

3. A production line consists of two machines in series. Machine 1 works uninterruptedly, there is always supply of new products. The production time of a product on machine 1 consists of two independent exponentially distributed phases: the first phase with mean 30 minutes, the second phase with mean 20 minutes. The production time of a product on machine 2 is exponentially distributed with a mean of 30 minutes. In between machine 1 and machine 2 there is an infinite buffer. As soon as machine 1 has finished a product, this product is placed into the buffer. As soon as machine 2 has finished a product, it takes the next product out of the buffer (if available). If at that moment the buffer is empty, machine 2 waits until machine 1 has finished the next product.

   a) Determine the fraction of time machine 2 is working.
   b) Determine the distribution of the sojourn time of a product at the second stage of the production line (i.e., the time period starting at the moment that a product enters the buffer and ending at the moment that the production time of the product at the second machine is completed).
   c) Determine the mean number of products on machine 2 and in the buffer between machine 1 and machine 2, respectively, at an instant a product is placed into the buffer.
   d) Determine the mean number of products on machine 2 and in the buffer between machine 1 and machine 2, respectively, at an arbitrary instant.

4. A machine produces products in two phases. The first phase is standard and the same for all products. The second phase is customer specific (the finishing touch). The first (resp. second) phase takes an exponential time with a mean of 10 (resp. 2) minutes. Orders for the production of one product arrive according to a Poisson stream with a rate of 3 orders per hour. Orders are processed in order of arrival.

   a) Determine the mean production lead time (waiting time plus production time) of an order.

   The machine is switched off when the system is empty and it is switched on again as soon as the first order arrives. A fixed cost of 20 euro is incurred each time the machine is switched on (the time needed to switch the machine on or off is negligible).

   b) Determine the average switch-on cost per hour.

   To reduce the production lead time one decides to start already with the production of phase 1 when the system is empty. If upon completion of phase 1 no order has arrived yet, the production stops and the machine is switched off. When the first order arrives the machine is switched on again and can directly start with phase 2.

   c) Determine the reduction in the mean production lead time.

Credits:

1a	b	c	2a	b	c	d	3a	b	c	d	4a	b	c
4	3	3	3	2	3	2	2	3	2	3	3	3	4