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Mobile Fulfillment Systems: Model and Design Insights

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Abstract The Mobile Shelf-based Order Pick (MSOP) system is a recent solution for automating pick, pack, and ship activities in distribution centers. In this system, the items are stored on movable storage shelves, also known as inventory pods, and brought to the order pick stations by robotic drive units. By using coordinated robots and decentralized decision making, the MSOP system has reported increased flexibility and productivity in order picking activities. We develop stylized performance evaluation models to analyze both order picking and replenishment processes in MSOP systems, based on multi-class closed queueing network models. We derive order throughput time performance for different choices of pod storage strategies in aisles, different depth-to-width ratios of the storage area using customized travel time expressions of the robots in the aisles. We also investigate whether a dedicated robot system for processing order picking and replenishment activities is beneficial over a pooled robot system. The models are validated using detailed simulations. Systems with many short aisles lead to shorter expected throughput times than systems with few but longer aisles. By using pooled robots instead of dedicated robots, the expected throughput time for order picking reduces upto one-third of its initial value; however, the expected replenishment time estimate increases upto three times.

Keywords Mobile shelves, order picking, pod storage policies, queueing models, design insights

1 Introduction

Warehouses are increasingly employing automation technologies to reduce operational costs, increase customer satisfaction and improve operational efficiency and productivity. A Mobile Shelf-based Order Pick (MSOP) system — a parts-to-picker order pick system — is a new paradigm for automating pick, pack and ship activities in distribution centers that significantly improves worker productivity and throughput capacity. The MSOP system, based on mobile-rack technology, is pioneered by Kiva Systems (Wurman et al. (2008); D'Andrea and Wurman (2008); Mountz (2012)). The items are stored on movable storage shelves (also referred as the inventory pods, see Figure 1). The product search and retrieval of the inventory pods are performed by small, autonomous drive-units (also referred as robots). The Kiva warehouse system is depicted in Figure 2.

Upto 55% of the operating costs at a distribution center are due to its order pick costs, which include costs associated with item picking, consolidation, and order packing (De Koster et al. (2007)). The traditional part-to-picker order pick systems such as the miniload AS/RS has three primary inefficiencies: 1) a sequential order picking flow (only one item can be picked simultaneously in an aisle); 2) not easily scalable and 3) inability to handle peak demands. These inefficiencies, which result in delivery time delays and high operating costs, can be potentially overcome by an MSOP system. In an MSOP system, movable storage shelves, which contain items ordered by a customer, are brought automatically to an order picker. Due to the large number of robots which can transport many item-pods simultaneously, an order picker can complete the order in a shorter time and can complete more orders in a day compared to traditional picker-to-parts systems (Enright and Wurman (2011)). An MSOP system provides added benefits of flexibility and scalability in addition to the associated advantages of automation. Most e-retailers have massive peaks in their distribution shipping volumes (during special occasions such as Thanksgiving or Christmas). By adding more robots, pods and/or work stations, the throughput capacity of handling additional orders can be addressed economically and in a relatively short time span (`http://www.mwpvl.com/html/kiva_systems.html`). Users of the Kiva Mobile-robotic Fulfillment System$^{\text{TM}}$

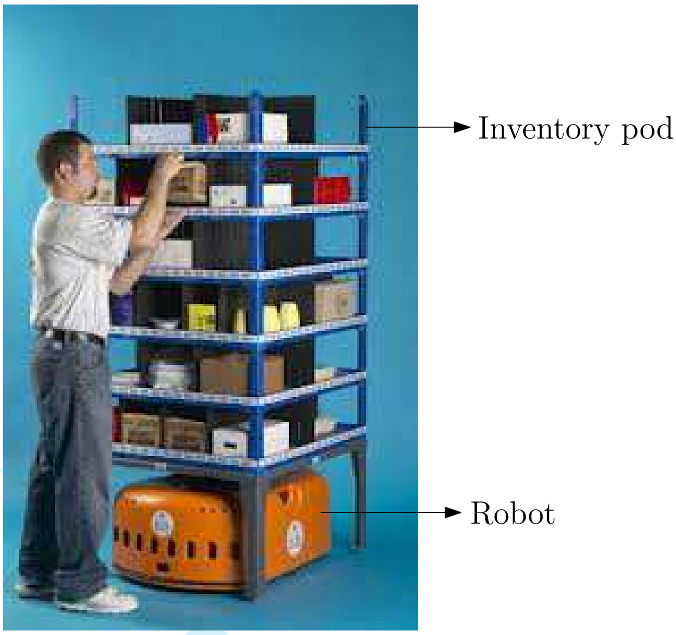


Figure 1: An inventory pod being carried by a Kiva robot (adapted from http://www.bastiansolutions.com/images/automation/kiva-unit.jpg)

Figure 2: Kiva warehouse system (source: http://www.kivasystems.com)

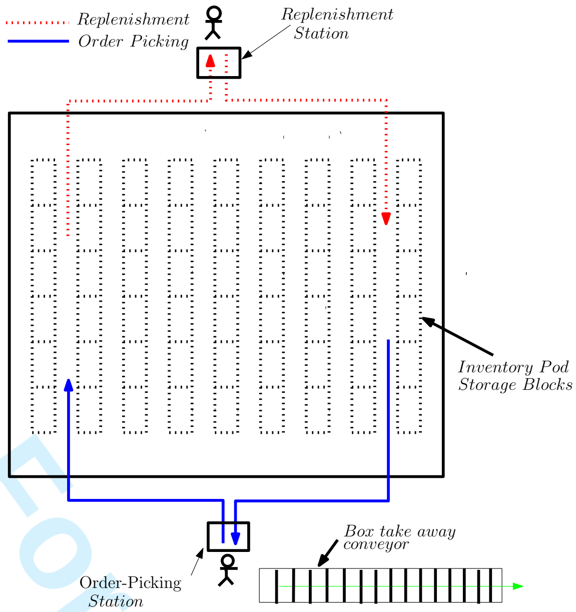


Figure 3: Typical configuration of Kiva system (adapted from Wurman et al. (2008))

include several established retailers such as Staples, Walgreens, Toys 'R' Us, Gap Inc., and Amazon. According to Enright and Wurman (2011), the delivery times and productivity of these retailers improved substantially with the Kiva system implementation.

Figure 3 shows typical flows within a Kiva system (Enright and Wurman (2011)). In this configuration, the picker prepares the shipping cartons and transfers them to the pack station using conveyors. The flow of the pod movements during the order picking and replenishment process is shown using solid and dotted lines, respectively. The location of a pod (carrying a particular item) is dynamic and is determined by the item turnover. The most frequently used items are stocked closer to the order pick station.

Several operational decisions affect the system throughput performance. For instance, the choice of pod storage location, the choice of order assignment to the order pick stations, and the choice of the robot to fetch the pod, affect the throughput performance. Likewise, the design choices such as the number of robots, depth-to-width ratio of the storage area, and maintaining a dedicated or a pooled fleet for order pick and replenishment processes may affect the system performance.

In this research, we specifically investigate the performance of the order pick system with different pod storage policies: “random open location storage” and “closest open location storage.” The random open location storage may seem to be the most efficient and commonly used storage policy, where any pod (after an order pick) is equally likely to be stored in any of the open locations. The primary objective of this policy is to maximize space utilization. In the closest open location storage policy, the pod is stored in the closest open location within the aisle that has been chosen for retrieval. The closest open location storage policy may be useful in reducing the travel time of the robot.

Further, robots may be pooled to perform both order pick and replenishment processes, where the robots complete an order pick operation followed by replenishment (if needed), in a so-called dual-command operation. It is not clear whether a dedicated fleet of robots performing single-command cycle operations (either order picking or replenishment) provides better throughput performance. Further, we also investigate the effect of the depth-to-width ratio on the system performance.

We address the following research questions:

- How many robots are required to meet the system throughput requirements?
- Do dedicated robots in order picking and replenishment (single command cycles) perform better than pooled robots, which are shared between order picking and replenishment?
- Which pod storage policy performs better, random open location (ROL) or closest open location (COL) storage?
- Which depth/width ratio results in better throughput performance?

The MSOP system is modeled as a closed queueing network with a simplification of the robot movement. The robots move only along the aisles and cross-aisles present in the order pick area. This robot movement differs from the original design of the Kiva System, where the empty robots can also travel underneath the pods to access another aisle. The model development follows a two-stage approach: 1) Markov-chain based models are developed to analyze the travel time within an aisle, 2) customized queueing network models are developed to analyze order picking and replenishment processes, and to answer each research question. To analyze the effect of alternative pod storage policies, we develop a single-class closed queueing network model. To analyze the effect of robot allocation (pooled vs dedicated), we develop two queueing network models with multiple robot classes. In the first multi-class closed queueing network model, the robots are pooled and they switch classes based on transaction probabilities (order picking vs replenishment) whereas in the second multi-class model, two dedicated fleets are used, one for order picking and another for replenishment. The model also captures the interference delays that could potentially occur in the aisles (if multiple robots attempt to access the aisle at the same time). We use a collision-avoidance protocol for the aisle movement. The models are evaluated using the approximate mean value analysis algorithm and validated with simulation. The analytical models present an attractive alternative to simulation for optimizing design parameters and improving system performance measures. The performance measures obtained from the models include robot utilization, system throughput and expected throughput time for order picking and replenishment.

The remainder of this paper is organized as follows. Section 2 reviews literature on order pick systems. Section 3 describes the system operations and the modeling assumptions used in this paper to develop the queueing network model. In Section 4, we describe the system and modeling assumptions. The queueing network models developed for the MSOP system with various layout configurations are presented in Section 5. The numerical results and the insights obtained from the analytical models are discussed in Section 6. Section 7 reports our conclusions and provides directions for future research.

2 Literature Review

In this section, we review literature in the area of parts-to-picker order pick systems with a specific focus on MSOP and vehicle-based warehouse systems. In all cases we restrict our review to travel-time and performance evaluation models, which is also the scope of our study.

Order picking is the most labour-intensive operation in warehouses and a very capital-intensive operation in warehouses with automated systems (De Koster et al. (2007)). Several travel time models for order picking systems that consider a specific equipment, storage policy and order picking area layout combinations are proposed in the literature. Daniels et al. (1998) develop an order picking model in which they determine the assignment of inventory to an order and the associated sequencing decisions in which the selected locations are visited. A detailed literature overview on typical design and control problems such as optimal layout design, storage assignment methods, routing methods, order batching and zoning in manual order picking processes is provided by De Koster et al. (2007).

Several different parts-to-picker systems can be distinguished such as the miniload AS/RS, MSOP systems, and vehicle-based tote handling systems. Our work is motivated by the Kiva systems in which movable storage shelves can be lifted by small autonomous robots Enright and Wurman (2011); D'Andrea (2012). A detailed overview of the Kiva system is given in Wulfraat (2012).

To the best of our knowledge, existing research on analyzing performance of Mobile Shelf-based Order Pick systems is limited. However, studies on performance analysis of autonomous vehicle-based warehouse systems are available. For instance, some papers discuss the estimation of cycle times for autonomous devices in Autonomous Vehicle-based Storage and Retrieval Systems (AVS/RS). An AVS/RS relies on autonomous vehicles to provide horizontal movement within a tier and uses lifts to provide vertical movement between tiers. Malmborg (2003a) and Malmborg (2003b) propose cycle-time estimation models based on Markov Chains. Fukunari and Malmborg (2008) develop an iterative computation algorithm based on queueing approximations to estimate the cycle times in AVS/RS. Fukunari and Malmborg (2009) extend the previous work by developing a queueing network approach to estimate the proportion of single and dual command cycles in the system. Kuo et al. (2007) model the movement of autonomous vehicles as an $M/G/V$ queue nested within an $M/G/L$ queue for estimating the expected travel times of vehicles and lifts. In such an AVS/RS, vehicle blocking delays in the aisles and cross-aisles can significantly impact system throughput and transaction cycle times. Roy et al. (2014) develop protocols to model vehicle blocking using a queueing network model and evaluate design trade-offs. A comprehensive review of collision prevention strategies in AGV systems can be found in Le-Anh and De Koster (2006).

Roy et al. (2012) develop a multi-class semi-open queueing network model to investigate several design decisions such as the configurations of aisles and columns, allocation of resources to zones, and vehicles assignment rules that significantly affect the performance of a system. Ekren et al. (2013) and Cai et al. (2013) develop semi-open queueing network models for analyzing the performance of the multi-tier AVS/RS with pooled vehicles. These models provide insights on the impact of tier configuration and sizing decisions on expected cycle times. However, note that existing studies on AVS/RS have been restricted to only pallet-based high density storage systems and not order pick systems.

Replenishment activities are very important to prevent shortage of products during intensive order picking (De Koster et al. (2007)). However, current literature on vehicle-based storage and retrieval systems do not explicitly model replenishment strategies. The performance of replenishment activities becomes more frequent and crucial during a busy picking periods (for example, Thanksgiving and Christmas). To the best of our knowledge, this research is a first attempt to model a mobile-racks based order pick system and to answer the design tradeoffs by considering both order picking and replenishment activities. Particularly, we investigate the effect of storage strategies and investigate the performance of dedicated and pooled robot configurations in the MSOP system.

3 System Operations and Modeling Assumptions

In this section, we describe the operations and the system modeling assumptions considered for an MSOP system. We then describe the protocol we develop for avoiding collisions of the robots in an aisle.

3.1 System operations

The system consists of an even number of aisles A. This simplifies the analysis, but it is straightforward to derive expressions for an odd number of aisles. We assume that an equal number of mobile shelves (pods) with same dimensions are present along the two sides of an aisle. A single pod is assumed to be l meter long and w meter wide. The order pick area is D meters long and W meters wide. The warehouse shape factor is characterized by depth/width, which is the ratio between D and W. D proxies the number of aisles and W proxies the number of pod storage locations per aisle. The two terms, d_{arr}^{dep} and d_{arr}^{loc} denote the distance between the departure and the arrival paths, and the distance between the arrival path and the first bay location, respectively.
To answer our research questions, we consider two different layouts of a warehouse: layout 1 with only an order pick station, and layout 2 with both an order pick and a replenishment station (shown in Figure 4).

Layout 1: We consider an order pick station located in the middle of the cross-aisle (in front of the aisles). This area is called the order pick area. A replenishment station is not considered in this layout. After completing an order pick, the robot first travels using the *departure path* of a cross-aisle and then travels along the center line of an aisle to store the pod in a pod storage location. Then the robot lifts a pod from another storage location from the same side of the aisle and brings it along the *arrival path* to the order pick station. Note that the robots can travel bidirectionally within an aisle. However, we allow only one robot to access the aisle at a time (which will be explained later). We analyze this layout using two different pod storage strategies.

Layout 2: We consider two areas, an order pick area and a replenishment area in a warehouse where the order pick station is in the order pick area and the replenishment station is in the replenishment area (opposite of the order pick area), see Figure 4. When the pod is emptied, a robot lifts the pod and travels to the replenishment station to replenish the pod. Using this layout, we later answer the trade-offs between usage of dedicated and pooled robots for order picking and replenishment.

A picker is stationed at the order pick station for picking items (filling orders) and a worker is stationed at the replenishment point for restocking the bins in the inventory pods. We assume that there is always a sufficient number of orders waiting to be processed by the robots, i.e. the robots never wait for order arrivals.

3.2 Assumptions

The following assumptions are made for the analysis of the MSOP system:

- The order pick station and the replenishment station have sufficient waiting space for robots (with pods).
- All storage locations and the pods have the same size. Therefore all storage locations are candidates for storing or retrieving any pod.
- Robots are scheduled according to the First-Come-First-Served (FCFS) policy.

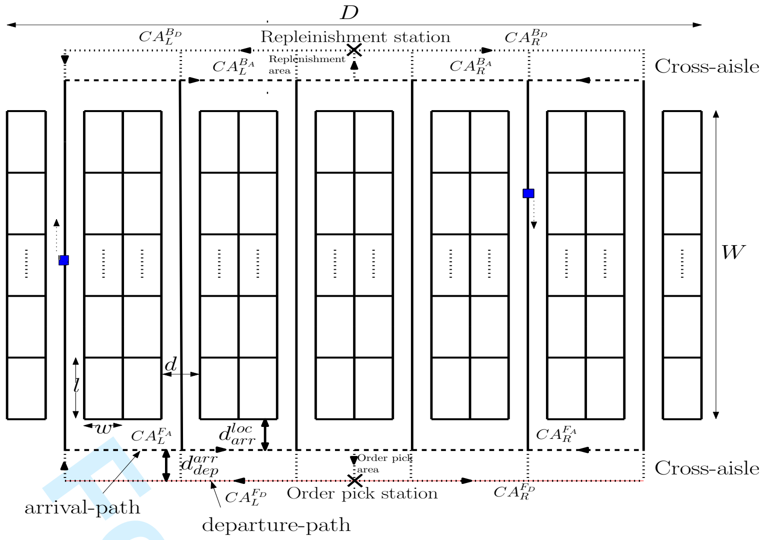


Figure 4: Layout of a warehouse with both order pick and replenishment station

- A dual-command cycle is considered for the storage and retrieval, i.e., a robot will perform a pod storage followed by a pod retrieval request in a single trip for both order picking and replenishment.
- The travel velocity of a robot is constant and robot acceleration/deceleration effects are ignored.
- To decide the next pod retrieval location, the robot chooses the left or right portion of the cross-aisle with equal probability. Further, each aisle and the pick side within an aisle for pod retrievals are chosen randomly.
- Each side of an aisle has at least one open location available for pod storage. Each robot stores a pod at the side of an aisle where the next retrieval has to be carried out. Note this is not a very restrictive assumption, as these storage systems work with many open spaces (20-30%), see Figure 2. We use this assumption to derive the aisle travel times with different pod storage strategies.
- All items for filling an order are present in one inventory pod.
- There are always orders waiting to be served by the robot, i.e., the queue of orders is never empty.

Note that most assumptions can be relaxed, we come back to that in the conclusions section.

3.3 Aisle Protocol for the MSOP System

In the MSOP system, blocking can occur during the movements of a robot in the aisles. That is, a robot in the process of storing and retrieving the pod in an aisle could be blocked by another robot entering the same aisle. For collision-avoidance during robot movements, it is assumed that a robot will only enter the aisle when the previous robot has completed its service and leaves the last point of the aisle.

4 Modeling the MSOP System

Using a queueing model, we analyze the effect of multiple design parameters on system performance measures. In this section, we describe the nodes in the queueing models and the service time distributions at the nodes.

4.1 Network Nodes

As discussed in Section 3 we consider two warehouse layouts. In the first layout, there is a departure and arrival path only in the order pick area. In the second layout, the departure and arrival paths are present in both the order pick (front) and the replenishment areas (back). The order pick (OP) station is located at the middle of the cross-aisle in the departure path. We divide the cross-aisle departure path into two equal segments ($CA_L^{F_D}$ and $CA_R^{F_D}$ corresponding to the left and right segment of the cross-aisle in the order pick area). Likewise, the arrival path along the cross-aisle is divided into two equal segments ($CA_L^{F_A}$ and $CA_R^{F_A}$ corresponding to the left and right segment of the cross-aisle in the arrival path). A similar cross-aisle segmentation is done for the departure and arrival paths at the replenishment station ($CA_L^{B_D}$ and $CA_R^{B_D}$ for the departure path and $CA_L^{B_A}$ and $CA_R^{B_A}$ for the arrival path). Each segment of the cross-aisle is modeled as an Infinite Server (IS) queue. A robot starts its service and accesses either the left or the right side of the segments on the departure path (with equal probability, 1/2). It then chooses any one of

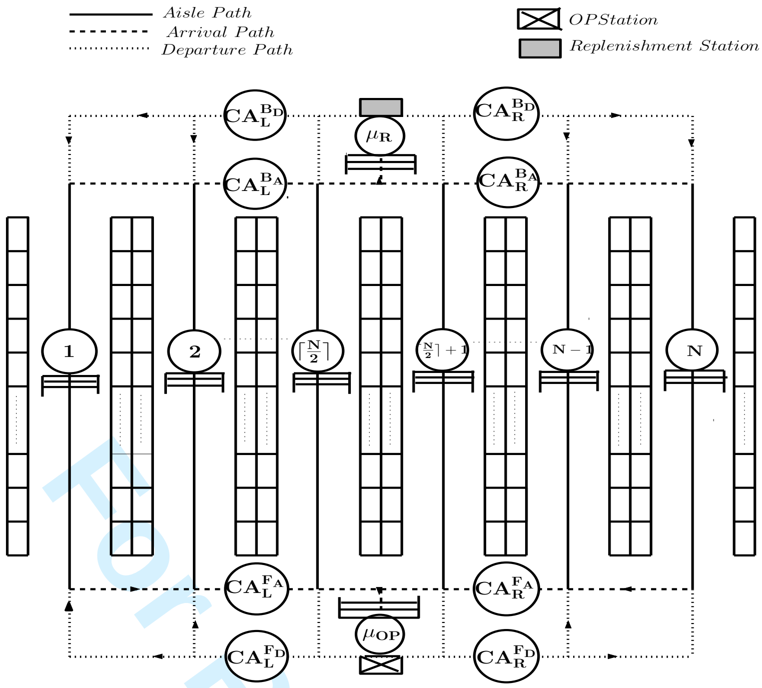


Figure 5: Queueing model description

the aisles with equal probability $p = \frac{1}{A/2}$, and then accesses either pick face of the aisle and moves towards an open rack location to store a pod (based on the pod storage strategy). Next it moves towards a pick-up location, retrieves the pod and exits the aisle. As soon a robot exits the aisle it accesses the arrival path segment based on its current position from either the left or the right side of the cross aisle.

Based on the aisle protocol, only one robot can enter the aisle at a time; therefore, each aisle is modeled as a single server queue with an infinite buffer size. The picking and the replenishment stations are modeled as single server queues with service rates μ_{OP} and μ_R respectively. Since we assume that orders are always waiting to be served by the robot, we model the layouts 1 and 2 as closed queueing networks (see Figure 5).

The stations in the network have general service times, which are characterized by the mean and the squared coefficient of variation (SCV).

We now present expressions for the first two moments of the service times describing the robot movements in the aisles and cross-aisles, and the fulfillment activities at the order pick and replenishment stations. To estimate the service times in aisles, we first determine the steady state probabilities of open locations at each side of an aisle for both pod storage strategies.

4.2 Aisle Service Time Estimation

We use two different pod storage strategies, random and closest open location, as described in Section 1. To obtain the mean and variance of the travel and handling time of a robot during storage and retrieval of a pod in an aisle, for both strategies, we describe the aisle by a discrete-time Markov chain. The status of storage locations in each side of an aisle can be either *open* or *used*. A location is denoted as open when there is no pod at that location and denoted as used when a pod is stored at that location. The state of an aisle can then be defined as a vector $\mathbf{s} = (a_1, a_2, \cdots, a_N)$, where a_k denotes the status of the storage location index k:

$$a_k = \begin{cases} 1, & \text{when location } k \text{ is used;} \\ 0, & \text{when location } k \text{ is open.} \end{cases} \tag{1}$$

Let N denote the total number of locations at each side of an aisle, out of which m locations are open. The state space S_m consists of all vectors $\mathbf{s}$ for which $a_1 + \cdots + a_N = N - m$. Hence $M = \binom{N}{m}$ is the number of states in S_m.

Under the random open location storage strategy, exactly $m(N-m)$ states can be reached by a robot (during pod storage and retrieval), with equal probability, from every state $\mathbf{s} \in S_m$. A state transition occurs when a robot stores a pod in an open location and retrieves a pod from an used location. Since for every state transition, the reverse transition is also feasible, we can immediately conclude that the Markov chain is double stochastic, and hence, the stationary distribution is uniform, i.e.,

$$\pi(\mathbf{s}) = \frac{1}{M}, \quad \forall \mathbf{s} \in S_m.$$

Note that this result also follows from symmetry.

Under the closest open location storage strategy the situation is different. Consider again a side of an aisle with N storage locations and m open locations. In this case, it is readily verified that the states $\mathbf{s}$ for which the last $m-1$ positions are open, i.e., $a_{N-m+1} = \cdots = a_N = 0$, for some $m \geq 1$, form an absorbing class of states, and this class can be reached from every state. Let $S_{a,m}$ denote this absorbing class of states. Once this set of states has been reached, the closest open location storage strategy reduces to the random open location storage strategy with 1 open location in an aisle with only $N-m+1$ locations, since the last $m-1$ locations in the aisle will never be used. Hence, the stationary distribution is given by

$$\pi(\mathbf{s}) = \frac{1}{N-m+1}, \quad \forall \mathbf{s} \in S_{a,m},$$

and $\pi(\mathbf{s}) = 0$ otherwise. Clearly, this indicates inefficient use of open locations.

4.3 Expectation and Variance of the Service Time in an Aisle

In this section, we determine the average service time in an aisle for the two pod storage policies. The robot service time in aisles includes the total travel time required to store a pod and to retrieve another pod, and the pod handling times (pick-up and set-down times).

Random open location storage policy: The service time in aisle, $t_{Aisle,R}$, for the random open location storage strategy will include the travel time of a robot to store a pod and retrieve another pod, including the return travel ($t_{WL,R}$), handling time to store a pod (t_{store}), handling time to retrieve a pod ($t_{retrieval}$) and the time associated to travel between the arrival-path (onward as well as return) of a cross-aisle to the starting point of the aisle location, d_{arr}^{loc} with a robot velocity v_r, which gives the following form:

$$t_{Aisle,R} = t_{WL,R} + t_{store} + t_{retrieval} + \frac{2d_{arr}^{loc}}{v_r} \tag{2}$$

Note that the travel time associated with the travel between the arrival and the departure paths is included in the cross-aisle service time (which will be explained later). Let i_s denote the location for storing a pod and i_r denote the location of retrieving a pod. Let $D_R(i_s, i_r)$ denote the distance travelled to store at location i_s and retrieve a pod from location i_r and return. In general, the distance travelled $D_R(i_s, i_r)$ for storing a pod at location i_s

and retrieving a pod from location i_r for the random open location storage strategy, and returning back to the same cross-aisle is given by the following expression:

$$D_R(i_s, i_r) = 2li_m - l \tag{3}$$

where $i_m = \max(i_s, i_r)$.

From the Markov chain analysis (of the random open location storage strategy), we can conclude that both i_s and i_r are uniform on the N locations (*independent* of the number of open locations). Therefore, the joint probability distribution of i_s and i_r is expressed as follows:

$$P(i_s = k, i_r = l) = \frac{1}{N(N-1)} \quad \forall k, l \in \{1, \dots, N\}, k \neq l \tag{4}$$

Using (4), the probability mass function, and the first and second moments of i_m are obtained using the following expressions.

$$P(i_m = k) = \frac{2(k-1)}{N(N-1)} \quad \text{where } k = 2, \dots, N \tag{5}$$

$$E[i_m] = \frac{2(N+1)}{3} \tag{6}$$

$$E[i_m^2] = 2(N+1)\left[\frac{N}{4} + \frac{1}{6}\right] \tag{7}$$

Using (3), (6), and (7), we can immediately obtain the first and the second moments of the distance travelled to store and retrieve a pod within the locations in an aisle, $E[D_R]$ and $E[D_R^2]$, using the following expressions.

$$E[D_R] = \frac{(4N+1)l}{3} \tag{8}$$

$$E[D_R^2] = \frac{(6N^2 + 2N - 1)l^2}{3} \tag{9}$$

The expected service time by a robot in the aisle can be obtained by taking the expectations of the terms in (2), which is given as:

$$E[t_{Aisle,R}] = E[t_{WL,R}] + t_{store} + t_{retrieval} + \frac{2d_{arr}^{loc}}{v_r} \tag{10}$$

Since $t_{WL,R} = \frac{D_R}{v_r}$, $E[t_{WL,R}]$ and $E[t_{WL,R}^2]$ are obtained from (11) and (12) as follows:

$$\begin{aligned} E[t_{WL,R}] &= \frac{(4N+1)l}{3v_r} && (11) \\ E[t_{WL,R}^2] &= \frac{(6N^2+2N-1)l^2}{3v_r^2} && (12) \end{aligned}$$

Therefore using (10), we obtain the service rate associated with the robot service time for the random open location storage strategy,

$$\mu_{Aisle,R} = \frac{1}{E[t_{Aisle,R}]} \tag{13}$$

Using (10) and (11), we obtain the expected service time $E[t_{Aisle,R}]$ of a robot in a aisle. The second moments of the robot service time in an aisle can be calculated as:

$$\begin{aligned} E[t_{Aisle,R}^2] &= E[t_{WL,R}^2] + \left(t_{store} + t_{retrieval} + \frac{2d_{arr}^{loc}}{v_r}\right)^2 \\ &+ 2\left(t_{store} + t_{retrieval} + \frac{2d_{arr}^{loc}}{v_r}\right) E[t_{WL,R}] && (14) \end{aligned}$$

The variance and squared coefficient of variation (SCV) of the robot service time within an aisle is given by

$$Var(t_{Aisle,R}) = E[t_{Aisle,R}^2] - E[t_{Aisle,R}]^2 \tag{15}$$

$$cv_{Aisle}^2 = \frac{Var(t_{Aisle,R})}{E[t_{Aisle,R}]^2} \tag{16}$$

Closest open location storage policy: Similar to the random open location storage policy, the service time in an aisle ($t_{Aisle,CL}$) for the closest open location storage strategy includes the traveling time within an aisle for storing and retrieving pods ($t_{WL,CL}$), handling times (t_{store} and $t_{retrieval}$), and time to travel the distance between the arrival-path of a cross-aisle to the starting point of the location d_{arr}^{loc}, which together gives the following form:

$$t_{Aisle,CL} = t_{WL,CL} + t_{store} + t_{retrieval} + \frac{2d_{arr}^{loc}}{v_r} \quad (17)$$

However, as discussed in Section 4.2, in steady state the closest open location storage policy for an aisle with given input parameters (N storage locations and m open locations) will have $m-1$ open locations towards the end of the aisle and one open location will lie in between the first and the $N-m+1$ storage locations. Therefore, the two moments of aisle travel time to store and retrieve a pod, including return travel, ($t_{WL,CL}$) in a closest open location storage strategy can be calculated using the same approach as for the random open location storage strategy considering a shorter aisle with $N-m+1$ storage locations and one open location.

4.4 Average Service Time for Cross-aisles

The cross-aisles are modeled as infinite server stations and hence the expected distance travelled by a robot on the cross-aisle on any of the eight cross-aisle segments (four in the order pick area and four in the replenishment area), $E[D_{CA}]$ can be obtained by the following expression:

$$E[D_{CA}] = \frac{A}{2}(w + \frac{d}{2}) + d_{arr}^{dep} \quad (18)$$

Using (18), the average travel time in a cross-aisle segment, $E[T_{CA}]$ can be obtained from

$$E[T_{CA}] = \frac{1}{v_r}E[D_{CA}] = \frac{1}{v_r}\left[\frac{A}{2}(w + \frac{d}{2}) + d_{arr}^{dep}\right] \quad (19)$$

where A is the total number of aisles.
The cross-aisle service rate is then expressed as:

$$\mu_{CA} = \frac{1}{E[T_{CA}]} \quad (20)$$

4.5 Service Time at Order Pick and Replenishment Station

The service time of the picker at the order pick station is given by t_{pick} and of the worker at the replenishment station is given by $t_{replenish}$, which are both deterministic times and therefore $E[t_{pick}] = t_{pick}$ and $E[t_{replenish}] = t_{replenish}$.

$$\mu_{OP} = \frac{1}{t_{pick}} \quad \text{and} \quad \mu_{Rep} = \frac{1}{t_{replenish}}. \tag{21}$$

We now describe the queueing network models of the MSOP system.

5 Queueing Network Models

In this section, we develop three closed queueing network models to answer the research questions. First, we analyze the effect of storage strategies on the MSOP system performance. For this question, we develop a single-class queueing network model with one order picking station. Second, our aim is to verify if dedicated robots for order picking and replenishment perform better than pooled robots in terms of throughput capacity. For this purpose, we propose two queueing network models with multi-class robots.

5.1 Queueing Network Model with Order-picking

In this subsection, we develop a closed queueing network model in which a single-class of V robots are considered for the order picking activity. The model is based on layout 1 described in Subsection 3.1. The queueing network shown in Figure 6 has $A + 5$ nodes, where A denotes the number of aisles. Node $A + 5$ represents the time spent by a robot at the order picking station, nodes $A + 1$ and $A + 2$ represent the time spent by the robot in the left and right side of the departure path in the cross-aisle, nodes $A + 3$ and $A + 4$ represent the time spent by the robot on the left and right side of the arrival path in the cross-aisle.

A robot begins the order pick activity by fetching a pod from the storage location and proceeds towards the arrival path of a cross-aisle and chooses $CA_L^{F_A}$ or $CA_R^{F_A}$. Then it proceeds towards the order picking station and waits in the buffer area for service at the order picking station (node $A + 5$). Then the robot chooses either a left or right node (i.e., $CA_L^{F_D}$ or $CA_R^{F_D}$) with equal probability. Thereafter, a robot chooses any of the $\frac{A}{2}$ aisles with probability $\frac{1}{A/2}$ and stores the pod based on a storage policy. In this model we consider two pod storage strategies, random and closest open location storage strategy. Using Approximate Mean Value Analysis (AMVA), we obtain the expected cycle time for order picking $E[CT_{op}]$, the throughput of order picking X_{op}, and expected queue lengths at various nodes.

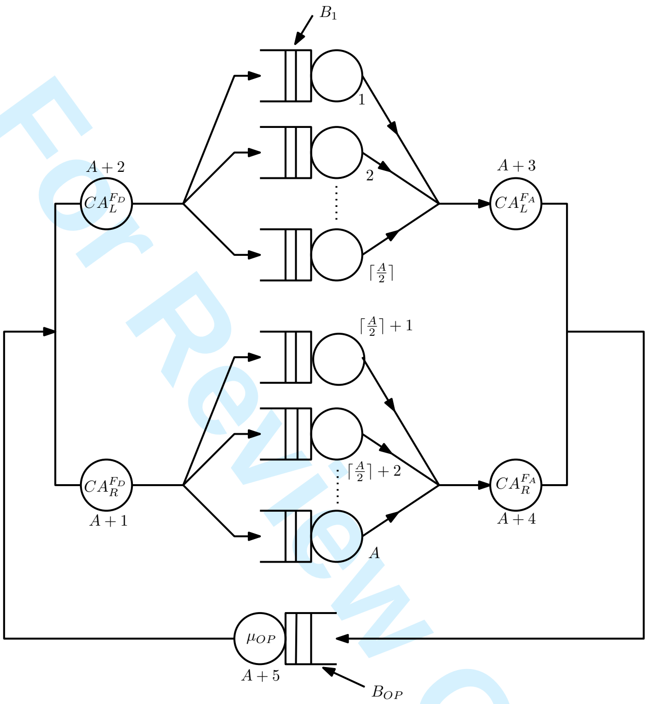


Figure 6: Single-class queueing network model for order picking using an MSOP system

5.2 Queueing Network Model with Order-picking and Replenishment

In this subsection, we develop a closed queueing network model in which two classes of robots, o and r, are considered for the order picking and replenishment station respectively. We denote the number of robots for each class as V_c, where $c \in \{o, r\}$. The network corresponding to Figure 7 consists of $A + 10$ service nodes, which denote the travel time in aisles, cross-aisles, order pick station, and replenishment station.

5.2.1 Queueing network model with dedicated robots

We first deploy two robot classes for the order pick and replenishment service, denoted by o and r respectively. The robots of class o which are deployed for the order pick service, begin their service by fetching a pod from an aisle storage location and travel back to access the cross-aisle nodes, either $A+5$ (i.e., $CA_L^{F_A}$) or $A+6$ (i.e., $CA_R^{F_A}$) (i.e., arrival-path of a cross aisle in forward direction), and proceed toward the node $A+1$ order pick station (see Figure 7). The robot waits in the order pick queue for its service (in a buffer area B_{OP}) and then accesses one of the two cross-aisle nodes, node $A+3$ (i.e.,$CA_L^{F_D}$) or node $A+4$ (i.e, $CA_R^{F_D}$) with equal probability, and then accesses any one of the $\frac{A}{2}$ aisle resources with probability $\frac{1}{A/2}$. The robot stores the pod in an open storage location in the aisle. Then the pod storage-retrieval cycle is repeated by the robot i.e., the robot moves towards a retrieval location, retrieves a pod from the same side of an aisle and proceeds to the order pick station. The picker takes t_{pick} time to fetch an item from the inventory pod. The movement of a class r, replenishment robot, is identical to the class o robot except that the class r robot is serviced at the replenishment station and it uses the cross-aisle resources in the replenishment area. Note that the aisle resources are shared by robots from both classes (see Figure 7). Similarly, a robot of class r, which is deployed for replenishment, fetches an inventory pod from a storage location, accesses either $A+9$ (i.e, $CA_L^{B_A}$) or $A+10$ (i.e., $CA_R^{B_A}$) based on its current location, proceeds towards the node $A+2$ (replenishment station), and waits in the queue for its service. The worker at replenishment station takes $t_{replenish}$ time to replenish a pod which causes the robots to wait in a buffer area B_R. After being served, the robot accesses one of the two cross-aisle nodes $A+7$ (i.e., $CA_L^{B_D}$) or $A+8$ (i.e., $CA_R^{B_D}$) (i.e., departure-path of cross-aisle in replenishment area) with probability 0.5, and then accesses any one of the $\frac{A}{2}$ aisles with probability $\frac{1}{A/2}$ to store the replenishment pod. This marks the end of one retrieval-storage replenishment cycle. After storing the pod, the robot moves towards a retrieval location, retrieves another replenishment pod, and the cycle repeats.

The movement of class r and class o robots are shown by dotted and solid lines respectively in Figure 7. Class switching is not allowed which means that the class o robots are not allowed to access the cross-aisle and replenishment station node of the replenishment area and the class r robots are not allowed to access the cross-aisle and order pick nodes of the order pick area in the queueing network model. Both classes of robots access the same aisle nodes because aisles are shared resources and only one robot can enter in an aisle at a time (refer the aisle protocol). Therefore, there is a buffer area, B_i, in front of aisle nodes, $i \in \{1, \ldots, A\}$.

Since both classes o and r access the same aisles to complete their service, the waiting times of a class o robots at aisle nodes is also dependent on the queue length of class r robots in front of the aisle.

Using Approximate Mean Value Analysis (AMVA) for multi-class closed queueing networks, we obtain the expected cycle times $E[CT_{op}]$ and $E[CT_{rep}]$, the class-specific expected queue lengths at all nodes, and the throughputs X_{op} and X_{rep} for the order pick and replenishment robot class, respectively (see Buitenhek et al. (2000) for details of the Approximate MVA algorithm). The basic AMVA algorithm is given below. The indices c, n_c, and k refer to the robot class, the number of robots in class c, and the node respectively. The terms $V^r_{c,k}$, $\bar{s}_{c,k}$, and $cv_{s_{c,k}}$ denote the visiting ratio, the average service time, and the service time coefficient of variation of class c robot at node k. The terms $Q_{c,k}(\vec{n})$, $U_{c,k}(\vec{n})$, $X_{c,k}(\vec{n})$, and $R_{c,k}(\vec{n})$ correspond to the average queue length, utilization, throughput, and residence time of class c robot at node k with $\vec{n}$ state of robots in the network.

Algorithm 1 AMVA Solution Algorithm for the Multi-class Queueing Network

Initialize $Q_{c,k}(\vec{0}) = 0$ for $c = 1, \cdots, C,\ k = 1, 2, \cdots, K$
Initialize $U_{c,k}(\vec{0}) = 0$ for $c = 1, \cdots, C,\ k = 1, 2, \cdots, K$
for $n = 1$ to V **do** ▷ *all states $\vec{n}$ for which $\sum_{c=1}^{C} n_c = n$ and $0 \le n_c \le V_c$, $V = \sum_{c=1}^{C} V_c$*
  for $c = 1$ to C **do**
    for $k = 1$ to K **do** ▷ *Residence Time Calculation*
      $R_{c,k}(\vec{n}) = \sum_{j=1}^{C} \left\{ U_{j,k}(\vec{n} - \vec{e}_j) \times \bar{s}_{j,k} \times \frac{(1+cv^2_{s_{j,k}})}{2} + (Q_{j,k}(\vec{n} - \vec{e}_j) - U_{j,k}(\vec{n} - \vec{e}_j))\bar{s}_{j,k} \right\} + \bar{s}_{c,k}$ ▷ *for Aisles, Order Pick, and Replenishment nodes*
      $R_{c,k}(\vec{n}) = \bar{s}_{c,k}$ ▷ *for Cross aisle nodes*
    end for
    $R_c = \sum_{k=1}^{K} V^r_{c,k} \times R_{c,k}(\vec{n})$ ▷ *Total residence time of class c*
  end for
  for $c = 1$ to C **do**
    for $k = 1$ to K **do**
      $X_c(\vec{n}) = \frac{n_c}{R_c}$ ▷ *Throughput of class c*
      $X_{c,k}(\vec{n}) = V^r_{c,k} \times X_c(\vec{n})$ ▷ *Throughput of class c at node k*
    end for
  end for
  for $k = 1$ to K **do**
    $X_k(\vec{n}) = \sum_{c=1}^{C} X_{c,k}(\vec{n})$ ▷ *Throughput at node k*
  end for
  for $c = 1$ to C **do**
    for $k = 1$ to K **do**
      $Q_{c,k}(\vec{n}) = X_{c,k}(\vec{n}) \times R_{c,k}$ ▷ *Calculate new queue lengths*
      $U_{c,k}(\vec{n}) = X_{c,k}(\vec{n}) \times \bar{s}_{c,k}$ ▷ *Calculate new utilizations*
    end for
  end for
end for

5.2.2 Queueing network model with pooled robots

A multiple class queueing network model with a single chain is developed for the MSOP system with pooled robots. Figure 8 illustrates a system with $A + 10$ nodes and 2 robot classes with class switching. The system operates in the following way. There are two robot classes, one for order picking and the other for replenishment service, denoted by class index o and r respectively. The robots of class o, which are deployed for the order picking, access one of the cross-aisle nodes, node $A+3$ (i.e.,$CA_L^{F_D}$) or node $A+4$ (i.e, $CA_R^{F_D}$) with probability 0.5, and the robots of class r, which are deployed for the replenishment station, accesses one of the cross-aisle nodes, node $A + 7$ (i.e.,$CA_L^{B_D}$) or node $A + 8$ (i.e, $CA_R^{B_D}$) with probability 0.5. Then a robot of any class l or r accesses any one of the $\frac{A}{2}$ aisles with probability $\frac{1}{A/2}$. Within an aisle, the robot first stores a pod and then moves toward a pod retrieval location. With a probability p_r, the retrieval request corresponds

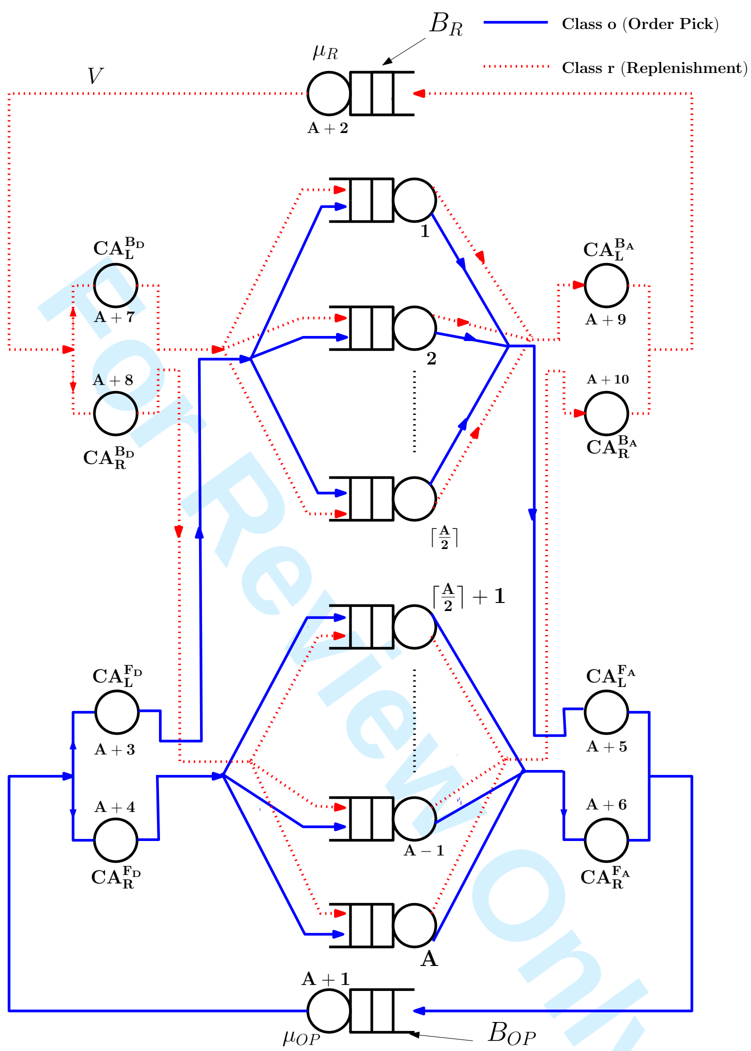


Figure 7: Queueing network model for the MSOP system with dedicated robot classes

to the order picking activity at the order pick station and with a probability $(1 - p_r)$, the retrieval request corresponds to the replenishment activity. Hence, probabilistic robot class switching is allowed in this queueing network model. The decision on the robot class switching should ideally be made immediately after the pod is stored in the storage location. After this service type decision, the robot either fetches an inventory pod for order picking or a replenishment pod for item replenishment. Since we model the complete aisle operation using a single service time distribution, we illustrate the class change operation in the Figure after the aisle service operation. Four cases are considered in which the robots either remains in the same class or switches to another class (see Figure 8).

1. If a class o robot receives a retrieval request for order picking (with probability p_r) then the class o robot remains in the same class o, retrieves another pod for order picking and then accesses the cross-aisle nodes either $A+5$ (i.e., $CA_L^{F_A}$) or $A+6$ (i.e., $CA_R^{F_A}$) (i.e., arrival-path of a cross aisle in order pick area) to reach the order pick station.

2. If a class o robot receives a retrieval request for replenishment activity (with probability $1 - p_r$) then after completing its service in aisle nodes, class o robot changes its class to class r robot, retrieves the replenishment pod, and accesses the cross-aisle nodes either $A + 9$ (i.e, $CA_L^{B_D}$) or $A + 10$ (i.e., $CA_R^{B_D}$) to reach the replenishment station.

3. If a class r robot receives a retrieval request for the replenishment activity (with probability $1 - p_r$) then the class r robot after completing a service in aisle nodes remains in the same class r, retrieves another replenishment pod, and accesses the cross-aisle nodes either $A + 9$ (i.e, $CA_L^{B_D}$) or $A + 10$ (i.e., $CA_R^{B_D}$) (i.e., arrival-path of a cross aisle in replenishment area), to reach the replenishment station.

4. If a class r robot receives a retrieval request for order picking (with probability p_r) then the class r robot after completing its service in aisle nodes, changes its class to class o robot, retrieves an inventory pod for order picking and accesses the cross-aisle nodes either $A+5$ (i.e., $CA_L^{F_A}$) or $A+6$ (i.e., $CA_R^{F_A}$) to reach the order pick station.

In this way, a single chain is formed in which the number of robots is constant in a chain; however, within the chain, the robots change their class depending on the type of processing request. After being serviced at the cross-aisle nodes, a robot with or without class switch will proceed towards the node $A+1$ (order pick station) or $A+2$ (replenishment station) with probability 1.

Note that both classes of robots access the same aisle nodes based on the developed aisle protocol and it is assumed that only one robot can enter an aisle at a time.

Since the robots can switch their class in the aisles for the retrieval request, the cycle times are associated with the following movements: a robot of any class starts its service by fetching a pod from a storage location, accesses the cross-aisles, service station (order pick or replenishment), accesses the cross-aisles to store the pod, and accesses the aisle to store the pod. Note that this cycle is equivalent if we consider the movement of the robot from either the order pick or the replenishment station in an aisle, travel for pod storage and another pod retrieval, and then return to either the order pick or the replenishment station to complete its service. Note that though the movement cycles are similar, the throughput time distributions in a cycle are different due to difference in the amounts of queueing at the nodes.

Using Approximate Mean Value Analysis (AMVA) for the multi-class closed queueing network with single chain, we obtain the expected cycle times $E[CT_{op}]$ and $E[CT_{rep}]$, the class-specific expected queue lengths at all nodes, and the throughputs X_{op} and X_{rep} for the order picking and replenishment robot class, respectively (Bolch et al. (2006)). We also obtain the expected queue lengths (Q_{OP}, Q_{Rep}, Q_{Aisle}) and utilizations (U_{OP}, U_{Rep}, U_{Aisle}) for the order pick, replenishment, and aisle resources respectively.

6 Numerical Experiments and Design Insights

To perform the numerical experiments, we use the system dimension data from the trade websites as well as from discussions with practitioners (see Table 1). We employ discrete-event simulations to validate the proposed analytical models for order picking and replenishment processes.

Simulation experiments are conducted with ARENA software with 8,000 time units (that corresponds to about 1 million order picks) and 15 replications were used to obtain the results. The confidence intervals for all performance measures are about ±2% of the average

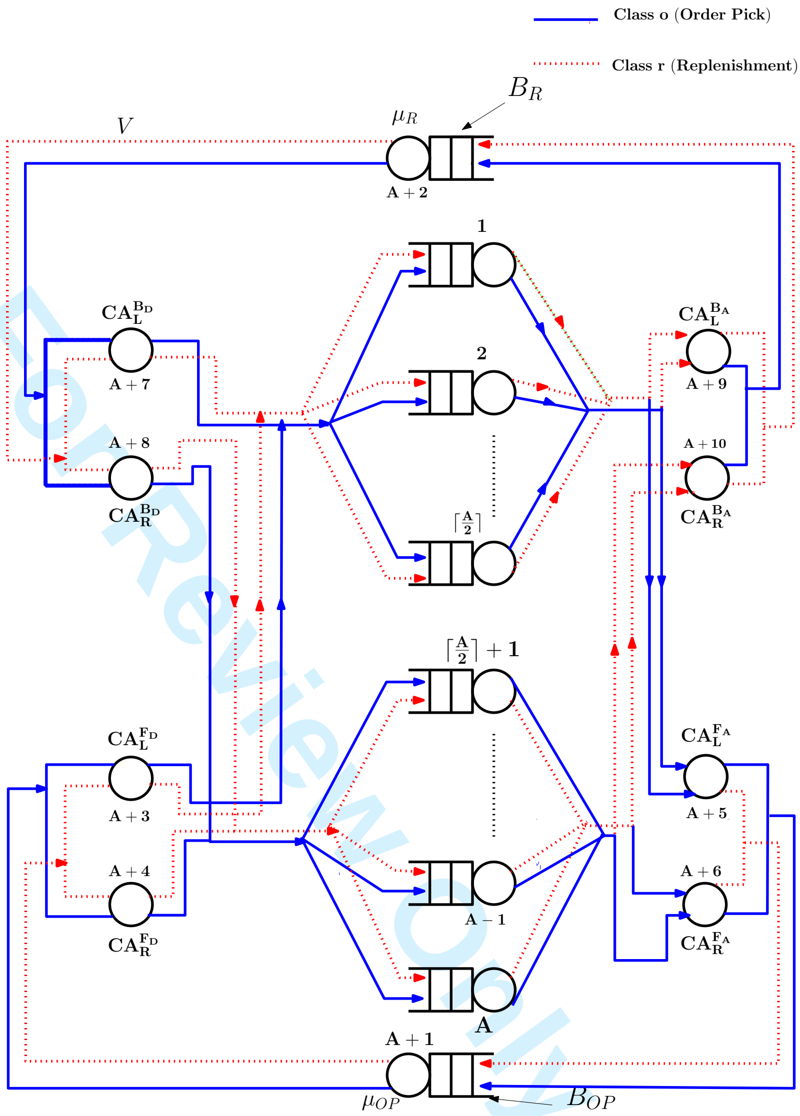


Figure 8: Queueing network model for the MSOP system with pooled robots

Table 1: System Dimensions and Operation Parameters

Symbol	Description	Values
d_{arr}^{dep}	Distance between arrival & departure path	1.2 meter
d_{arr}^{loc}	Distance between arrival path & starting point of rack locations	1 meter
$\frac{d}{2}$	Distance among each side of rack locations to aisle	1 meter
l	Gross length of pod location	0.99 meter
w	Gross width of pod location	1 meter
v_r	Speed of a robot	3 meter/sec
t_{store}	Time needed to store a pod	5 sec
$t_{retrieval}$	Time needed to retrieve a pod	5 sec
t_{pick}	Time needed for order picking at OP station	15 sec
$t_{replenish}$	Time needed for replenishing a pod	90 sec
p_r	Probability of an order pick transaction	0.8

measure. Robots decide to access either the left or the right cross-aisles with probability $\frac{1}{2}$. Then a robot uses any one of the $\frac{A}{2}$ aisles with probability $\frac{1}{A/2}$. To access an aisle a robot has to wait for the preceding robot to complete its service. As soon a preceding robot exits the aisle, the next robot enters an aisle to complete its service.

Table 2: Performance Measures of MSOP system using a single robot class

Input				Output: Performance Measure																	
Policy	N_s	$\frac{D}{W}$	V	$A_{Q_{OP}}$	$S_{Q_{OP}}$	$\delta_{Q_{OP}}$	$A_{Q_{Aisle}}$	$S_{Q_{Aisle}}$	$\delta_{Q_{Aisle}}$	$A_{U_{OP}}$	$S_{U_{OP}}$	$\delta_{U_{OP}}$	$A_{U_{Aisle}}$	$S_{U_{Aisle}}$	$\delta_{U_{Aisle}}$	$A_{E[CT_{op}]}$	$S_{E[CT_{op}]}$	$\delta_{E[CT_{op}]}$	A_X	S_X	δ_X
Closest	200	1	3	1.74	1.67	4.1%	0.18	0.18	1.9%	86%	88%	1.8%	0.17	0.17	1.5%	52.10	51.32	1.5%	207.36	210.43	1.5%
			4	2.59	2.59	0.1%	0.21	0.20	3.1%	94%	96%	1.6%	0.18	0.18	1.7%	63.51	62.31	1.9%	226.80	231.09	1.9%
			5	3.52	3.52	0.1%	0.22	0.22	1.4%	98%	99%	1.0%	0.19	0.18	5.5%	76.54	75.32	1.6%	235.08	238.97	1.6%
		2	3	1.85	1.83	1.3%	0.09	0.09	0.0%	89%	91%	1.8%	0.09	0.08	6.8%	50.36	49.26	2.2%	214.56	219.26	2.1%
			4	2.75	2.74	0.4%	0.10	0.09	9.2%	96%	98%	1.6%	0.09	0.09	2.3%	62.25	61.11	1.9%	231.48	235.65	1.8%
			5	3.71	3.70	0.3%	0.10	0.10	1.5%	99%	100%	1.1%	0.09	0.09	5.0%	75.80	74.55	1.7%	237.60	241.44	1.6%
Random	200	1	3	1.70	1.65	2.8%	0.19	0.19	0.4%	85%	87%	1.9%	0.17	0.17	2.3%	52.70	51.43	2.5%	204.84	209.98	2.4%
			4	2.53	2.53	0.1%	0.22	0.21	2.6%	94%	95%	1.2%	0.19	0.19	0.6%	63.92	62.22	2.7%	225.36	231.44	2.6%
			5	3.45	3.43	0.6%	0.23	0.22	3.6%	98%	98%	0.3%	0.20	0.19	4.7%	76.78	75.33	1.9%	234.36	238.94	1.9%
		2	3	1.81	1.80	0.6%	0.09	0.09	5.1%	88%	89%	0.6%	0.09	0.08	12.1%	50.89	49.81	2.2%	212.40	216.84	2.0%
			4	2.70	2.71	0.5%	0.10	0.10	3.9%	96%	97%	1.1%	0.10	0.09	8.1%	62.56	61.98	0.9%	230.04	232.32	1.0%
			5	3.65	3.67	0.5%	0.11	0.10	7.7%	99%	100%	1.3%	0.10	0.10	0.2%	75.95	74.98	1.3%	236.88	240.07	1.3%
Closest	400	1	3	1.57	1.53	2.7%	0.15	0.15	3.2%	82%	83%	0.7%	0.14	0.14	3.0%	54.60	53.21	2.6%	197.64	202.96	2.6%
			4	2.37	2.33	1.8%	0.18	0.17	4.1%	92%	94%	1.9%	0.16	0.16	0.8%	65.10	64.59	0.8%	221.04	222.93	0.8%
			5	3.27	3.17	3.0%	0.19	0.18	4.8%	97%	98%	1.1%	0.17	0.16	6.0%	77.41	76.48	1.2%	232.56	235.35	1.2%
		2	3	1.65	1.62	1.8%	0.11	0.11	1.6%	85%	86%	1.6%	0.11	0.10	5.9%	53.19	52.08	2.1%	203.04	207.36	2.1%
			4	2.48	2.46	0.9%	0.13	0.12	4.8%	94%	95%	1.3%	0.12	0.11	6.6%	64.02	63.24	1.2%	225.00	227.70	1.2%
			5	3.41	3.40	0.2%	0.13	0.13	1.9%	98%	99%	1.2%	0.12	0.12	2.0%	76.71	75.15	2.1%	234.72	239.51	2.0%
Random	400	1	3	1.54	1.51	1.7%	0.16	0.15	6.3%	81%	83%	1.9%	0.15	0.14	6.1%	55.26	53.80	2.7%	195.48	200.76	2.6%
			4	2.32	2.33	0.4%	0.18	0.18	1.9%	92%	93%	1.6%	0.17	0.16	4.3%	65.57	65.55	0.0%	219.60	219.70	0.0%
			5	3.21	3.15	1.8%	0.20	0.19	3.3%	97%	97%	0.5%	0.18	0.17	3.6%	77.70	77.48	0.3%	231.48	232.30	0.4%
		2	3	1.61	1.59	1.4%	0.12	0.11	5.3%	84%	85%	1.6%	0.11	0.10	9.7%	53.80	52.97	1.6%	200.88	203.89	1.5%
			4	2.43	2.44	0.3%	0.13	0.13	0.8%	93%	95%	2.0%	0.12	0.12	1.8%	64.43	63.44	1.6%	223.56	227.00	1.5%
			5	3.35	3.35	0.1%	0.14	0.13	6.5%	97%	99%	1.5%	0.13	0.12	6.5%	76.95	75.18	2.4%	234.00	239.43	2.3%

Table 3: Performance Measure of MSOP system with dedicated robots

Input: $\frac{D}{W}$, N_s, V — Output:Performance Measure (Dedicated Class)

$\frac{D}{W}$	N_s	V	$A_{Q_{OP}}$	$S_{Q_{OP}}$	$\delta_{Q_{OP}}$	$A_{Q_{Rep}}$	$S_{Q_{Rep}}$	$\delta_{Q_{Rep}}$	$A_{Q_{Aisle}}$	$S_{Q_{Aisle}}$	$\delta_{Q_{Aisle}}$	$A_{U_{OP}}$	$S_{U_{OP}}$	$\delta_{U_{OP}}$	$A_{U_{Rep}}$	$S_{U_{Rep}}$	$\delta_{U_{Rep}}$	$A_{U_{Aisle}}$	$S_{U_{Aisle}}$	$\delta_{U_{Aisle}}$
1	200	6	3.41	3.40	0.2%	0.78	0.77	1.7%	0.25	0.25	0.9%	96%	96%	0.5%	78%	77%	1.7%	22%	22%	1.4%
		8	5.34	5.34	0.0%	0.78	0.77	1.6%	0.27	0.26	2.3%	100%	99%	0.5%	78%	78%	0.3%	23%	22%	4.2%
		10	6.31	6.33	0.3%	1.72	1.72	0.0%	0.28	0.27	3.1%	100%	100%	0.0%	99%	99%	0.1%	24%	23%	3.3%
		14	9.29	9.31	0.2%	2.71	2.71	0.1%	0.28	0.28	1.4%	100%	100%	0.0%	100%	100%	0.0%	24%	24%	0.0%
2	200	6	3.56	3.59	0.8%	0.80	0.80	0.3%	0.12	0.12	0.5%	97%	99%	1.5%	80%	80%	0.3%	11%	10%	10.0%
		8	5.52	5.49	0.5%	0.80	0.80	0.2%	0.12	0.12	3.5%	100%	100%	0.0%	80%	80%	0.2%	12%	11%	4.6%
		10	6.51	6.53	0.4%	1.75	1.76	0.5%	0.12	0.12	3.3%	100%	100%	0.0%	100%	99%	1.0%	12%	11%	8.1%
		14	9.50	9.52	0.3%	2.74	2.75	0.3%	0.13	0.13	0.4%	100%	100%	0.0%	100%	100%	0.0%	12%	11%	9.0%
1	400	6	3.16	3.10	1.9%	0.75	0.75	0.5%	0.22	0.20	9.0%	95%	97%	2.4%	75%	74%	1.9%	20%	19%	2.9%
		8	5.06	5.03	0.6%	0.75	0.76	0.9%	0.23	0.20	14.4%	99%	98%	0.7%	75%	75%	0.4%	20%	20%	1.5%
		10	6.03	6.02	0.2%	1.68	1.68	0.2%	0.24	0.23	4.5%	100%	100%	0.5%	98%	96%	1.8%	21%	20%	5.6%
		14	9.00	8.99	0.1%	2.66	2.67	0.4%	0.25	0.23	6.9%	100%	100%	0.0%	100%	100%	0.0%	21%	20%	7.2%
2	400	6	3.28	3.30	0.7%	0.77	0.77	0.3%	0.15	0.14	9.8%	96%	98%	2.5%	77%	77%	0.3%	14%	13%	9.2%
		8	5.20	5.21	0.2%	0.77	0.77	0.3%	0.16	0.16	0.1%	99%	99%	0.2%	77%	76%	1.0%	15%	14%	4.9%
		10	6.18	6.20	0.4%	1.70	1.70	0.0%	0.17	0.16	4.6%	100%	100%	0.1%	98%	96%	2.5%	15%	15%	1.6%
		14	9.16	9.17	0.2%	2.69	2.60	3.3%	0.17	0.17	0.1%	100%	100%	0.0%	100%	100%	0.0%	15%	14%	10.3%

$\frac{D}{W}$	N_s	V	$A_{E[CT_{op}]}$	$S_{E[CT_{op}]}$	$\delta_{E[CT_{op}]}$	$A_{E[CT_{rep}]}$	$S_{E[CT_{rep}]}$	$\delta_{E[CT_{rep}]}$	$A_{X_{op}}$	$S_{X_{op}}$	$\delta_{X_{op}}$	$A_{X_{rep}}$	$S_{X_{rep}}$	$\delta_{X_{rep}}$
1	200	6	77.8	76.8	1.2%	114.9	112.3	2.3%	231.5	234.3	1.2%	31.3	32.1	2.3%
		8	105.5	104.2	1.2%	115.1	114.2	0.7%	239.0	241.8	1.1%	31.3	31.5	0.6%
		10	119.8	121.4	1.3%	181.7	189.3	4.0%	240.5	237.2	1.4%	39.6	38.0	4.1%
		14	163.7	165.7	1.2%	262.6	265.4	1.1%	242.1	239.0	1.3%	41.0	40.7	0.9%
2	200	6	76.9	77.5	0.7%	112.9	111.8	0.9%	230.4	232.3	0.8%	32.0	32.2	0.5%
		8	104.9	105.4	0.5%	112.2	113.3	1.0%	241.2	239.0	0.9%	32.0	31.8	0.9%
		10	119.4	121.0	1.3%	179.8	189.9	5.3%	230.4	238.0	3.2%	37.4	37.9	1.2%
		14	163.4	165.2	1.1%	261.5	263.5	0.7%	242.3	239.7	1.1%	41.4	41.0	1.0%
1	400	6	79.2	77.2	2.6%	119.3	120.3	0.8%	227.2	233.1	2.5%	30.2	29.9	1.1%
		8	106.4	108.8	2.3%	119.5	120.5	0.8%	236.9	231.6	2.3%	30.1	29.9	0.8%
		10	120.6	121.6	0.9%	184.3	190.9	3.5%	239.0	236.8	0.9%	39.2	37.7	4.1%
		14	164.1	166.7	1.5%	264.2	278.9	5.3%	241.2	237.6	1.5%	41.0	38.7	6.0%
2	400	6	78.5	78.2	0.4%	117.2	119.2	1.7%	229.3	230.1	0.3%	30.6	30.2	1.3%
		8	105.9	106.2	0.3%	117.3	119.4	1.8%	238.0	237.2	0.3%	30.6	30.1	1.5%
		10	120.2	121.3	0.9%	182.9	191.2	4.3%	239.8	237.4	1.0%	39.2	37.7	4.2%
		14	163.9	161.7	1.4%	263.3	264.3	0.4%	241.6	245.0	1.4%	41.0	40.9	0.4%

Table 4: Performance Measure of MSOP system with pooled robots

Input: $\frac{D}{W}$, N_s, V — Output: Performance Measure (Pooled Class)

$\frac{D}{W}$	N_s	V	$A_{Q_{OP}}$	$S_{Q_{OP}}$	$\delta_{Q_{OP}}$	$A_{Q_{Rep}}$	$S_{Q_{Rep}}$	$\delta_{Q_{Rep}}$	$A_{Q_{Aisle}}$	$S_{Q_{Aisle}}$	$\delta_{Q_{Aisle}}$	$A_{U_{OP}}$	$S_{U_{OP}}$	$\delta_{U_{OP}}$	$A_{U_{Rep}}$	$S_{U_{Rep}}$	$\delta_{U_{Rep}}$	$A_{U_{Aisle}}$	$S_{U_{Aisle}}$	$\delta_{U_{Aisle}}$
1	200	6	1.34	1.30	2.7%	3.31	3.40	2.7%	0.19	0.19	0.5%	67%	65%	2.8%	99%	98%	1.0%	17%	17%	0.1%
		8	1.58	1.50	5.3%	5.02	5.19	3.3%	0.20	0.19	3.9%	69%	66%	3.8%	99%	98%	1.0%	17%	17%	2.6%
		10	1.71	1.69	0.9%	6.89	6.90	0.2%	0.20	0.19	4.5%	69%	65%	5.7%	100%	100%	0.0%	18%	17%	2.9%
		14	1.77	1.69	5.0%	10.83	10.65	1.7%	0.20	0.19	3.7%	68%	66%	3.4%	100%	100%	0.0%	17%	17%	2.2%
2	200	6	1.36	1.31	3.8%	3.41	3.50	2.6%	0.09	0.09	0.0%	67%	64%	5.0%	94%	96%	2.2%	8%	8%	0.0%
		8	1.60	1.48	7.8%	5.14	5.25	2.0%	0.09	0.09	3.1%	69%	66%	4.0%	99%	98%	1.0%	9%	8%	8.9%
		10	1.71	1.69	1.4%	7.02	7.10	1.1%	0.09	0.09	3.3%	69%	66%	4.1%	100%	100%	0.0%	9%	8%	9.0%
		14	1.78	1.78	0.2%	10.97	10.88	0.8%	0.09	0.09	2.7%	68%	66%	3.4%	100%	100%	0.0%	9%	8%	8.2%
1	400	6	1.29	1.20	7.6%	3.12	3.15	0.9%	0.17	0.16	3.5%	66%	66%	0.2%	99%	99%	0.2%	15%	15%	0.5%
		8	1.55	1.51	2.7%	4.80	4.81	0.3%	0.17	0.17	1.5%	68%	63%	8.5%	99%	99%	0.0%	16%	15%	3.9%
		10	1.69	1.62	4.4%	6.64	6.66	0.3%	0.17	0.17	2.2%	69%	68%	1.0%	100%	100%	0.0%	16%	15%	4.5%
		14	1.77	1.73	2.5%	10.57	10.60	0.3%	0.17	0.17	1.6%	68%	68%	0.5%	100%	100%	0.0%	16%	15%	3.8%
2	400	6	1.31	1.30	0.9%	3.20	3.21	0.3%	0.12	0.11	6.1%	67%	66%	0.8%	100%	97%	2.8%	11%	10%	9.0%
		8	1.57	1.51	3.6%	4.90	4.91	0.3%	0.12	0.12	0.7%	68%	66%	3.7%	100%	99%	1.0%	11%	11%	2.0%
		10	1.70	1.59	6.9%	6.75	6.79	0.5%	0.12	0.12	1.2%	69%	66%	4.1%	100%	100%	0.0%	11%	11%	2.4%
		14	1.77	1.73	2.6%	10.69	10.70	0.1%	0.12	0.12	0.5%	68%	68%	0.4%	100%	100%	0.0%	11%	11%	1.7%

$\frac{D}{W}$	N_s	V	$A_{E[CT_{op}]}$	$S_{E[CT_{op}]}$	$\delta_{E[CT_{op}]}$	$A_{E[CT_{rep}]}$	$S_{E[CT_{rep}]}$	$\delta_{E[CT_{rep}]}$	$A_{X_{op}}$	$S_{X_{op}}$	$\delta_{X_{op}}$	$A_{X_{rep}}$	$S_{X_{rep}}$	$\delta_{X_{rep}}$
1	200	6	54.4	54.7	0.7%	321.5	323.9	0.7%	331.2	329.0	0.7%	11.2	11.1	0.7%
		8	59.1	59.5	0.7%	464.1	476.8	2.7%	426.3	423.4	0.7%	7.8	7.5	2.8%
		10	61.8	60.9	1.6%	625.9	638.7	2.0%	465.8	473.1	1.5%	11.5	11.3	2.1%
		14	63.6	65.3	2.6%	976.0	966.8	1.0%	622.9	606.8	2.6%	11.1	11.2	0.9%
2	200	6	52.3	51.3	2.1%	326.2	349.9	6.8%	343.9	351.2	2.1%	11.0	10.3	7.2%
		8	56.9	54.3	4.9%	471.8	485.6	2.8%	442.8	464.5	4.7%	7.6	7.4	2.9%
		10	59.5	56.6	5.0%	635.2	682.3	6.9%	484.4	508.5	4.7%	11.3	10.6	7.4%
		14	61.1	61.8	1.1%	986.4	987.2	0.1%	648.6	641.3	1.1%	10.9	10.9	0.1%
1	400	6	58.1	60.2	3.5%	311.9	315.3	1.1%	309.8	298.9	3.6%	11.5	11.4	1.1%
		8	63.1	66.2	4.8%	449.9	471.2	4.5%	399.6	380.4	5.0%	8.0	7.6	4.8%
		10	66.0	71.2	7.3%	609.2	621.3	1.9%	436.3	404.3	7.9%	11.8	11.6	2.0%
		14	68.0	69.2	1.7%	957.7	960.3	0.3%	582.2	572.3	1.7%	11.3	11.2	0.3%
2	400	6	56.4	57.3	1.5%	315.5	321.7	1.9%	318.9	314.0	1.6%	11.4	11.2	2.0%
		8	61.3	59.1	3.6%	456.0	466.8	2.3%	411.3	426.2	3.5%	7.9	7.7	2.4%
		10	64.1	61.5	4.2%	616.7	624.0	1.2%	449.4	468.4	4.1%	11.7	11.5	1.2%
		14	66.0	65.2	1.1%	966.2	963.2	0.3%	600.3	607.0	1.1%	11.2	11.2	0.3%

The error in percentage deviations, δ_{P_k} of performance measure P at node k, with respect to the simulation and analytical methods are calculated by the following expression:

$$\delta_{P_k} = \left| \frac{A_{P_k} - S_{P_k}}{S_{P_k}} \right| \times 100 \tag{22}$$

where A_{P_k} is the value of the performance measure, P at node k obtained from the analytical model and S_{P_k} is the value of the performance measure, P at node k obtained from the simulation model. A summary of input parameters for the experiments used to validate the model is presented in Table 1 and Table 2. Note that the $\frac{D}{W}$ ratio is set at two levels: 1 and 2; the number of aisles (A) is set at two levels: 8 and 10, the number of storage locations (N_S) is set at two levels: 200 and 400; and the number of drive units (robots, V) is varied at three levels: 3, 4, and 5 in the single-class model (with only order picking) and 6, 8, 10, and 14 in the multi-class model (with both order picking and replenishment). In all cases the simulation model took about 15 minutes to run on a high configuration PC. In contrast, the analytical model took less than 15 to 20 seconds to run for the same configuration. Hence, the analytical models can be very useful for quickly exploring various alternatives in the large design space of a warehouse.

The maximum and the average percentage errors for all performance measures including queue lengths, resource utilization, throughput, throughput times is about 10% and <5% respectively. The errors are higher in some occasions because the absolute value of the performance measure is itself very small.

In the next section, we present design insights based on the analysis of the three queueing network models.

6.1 Insights for the Order Picking Queueing Network Model

For better exposition, we first compare the performance of the order pick system by storage policy (random and closest open location storage strategy). It is expected that the closest open location storage strategy would take less residence time in the aisles than the random storage. The analysis results (provided in Table 2) shows that order pick operations with closest open location storage policy takes about 1%-2% less residence time than the random open location storage policy. Therefore, a system can gain a slightly higher throughput capacity with the closest open location storage policy than random open location storage policy.

To study the effect of the number of storage locations, we fix the $\frac{D}{W}$ ratio of the storage area, storage strategy and number of robots V. It is expected that the travel times within aisles (and hence expected throughput times) increase with an increase in the number of storage locations (Table 2 shows that the cycle times $E[CT_{op}]$ increase when the number of locations increase from 200 to 400), which results in the reduction in system throughput X.

We analyze the $\frac{D}{W}$ ratio which affects the performance of the MSOP based warehouse system. With increase in the $\frac{D}{W}$ ratio, the number of aisles increases. Therefore, the queue lengths in front of aisles decrease and the waiting times reduce. However, the effect of overall delay in aisles and cross-aisle is not clear because by increasing the number of aisles, the service times on the cross-aisle also increase. In Table 2, we note that when the ratio of $\frac{D}{W}$ increases from 1 to 2, the robots take 1-2% less cycle times and warehouse throughput per hour increase. In the case of random storage, with $N_s = 200$ locations and $V = 3$ robots, the throughput of a system X increases from 210 to 217 picks/hr when the $\frac{D}{W}$ increases from 1 to 2.

It is expected that the expected queue lengths Q_{Aisle} at the aisles and the order pick station Q_{OP} increases with the increase in the number of robots. This observation can be confirmed by the results given in Table 2. In a closest open location storage policy, with a fixed number of storage locations (200), $\frac{D}{W}$ ratio=1, and 86-98% utilization of the order pick station, it can be observed that by increasing the robots from 3 to 4 and then 5, the queue lengths are also increasing at aisles and at order pick station which affects the cycle time $E[CT_{op}]$ of a robot.

The effect of increasing the percentage of open locations has been found on throughput and utilization in the random and closest open location storage policy. By considering a fixed number of robots, in Figure 9, we observe that when the percentage of open locations increases from 10-90%, the throughput in the random open location storage strategy remains same throughout but the throughput increases in the closest open location storage strategy. Since in the random storage, the service time (for any number of open locations) is equal to the service time for two (one storage and one retrieval) randomly chosen locations and therefore the throughput does not change as the number of open location increases. Similarly, for the closest location, the two random locations are chosen from an aisle with

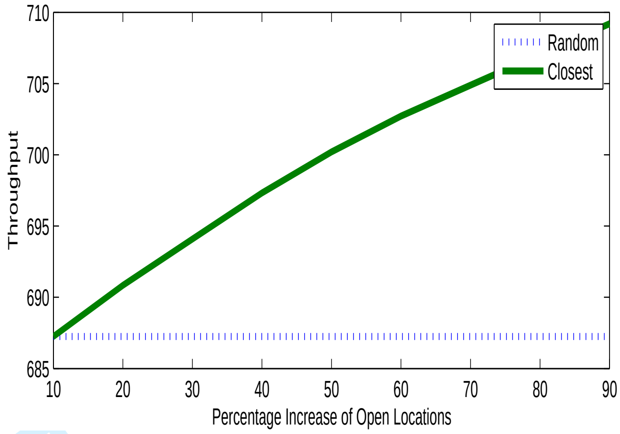


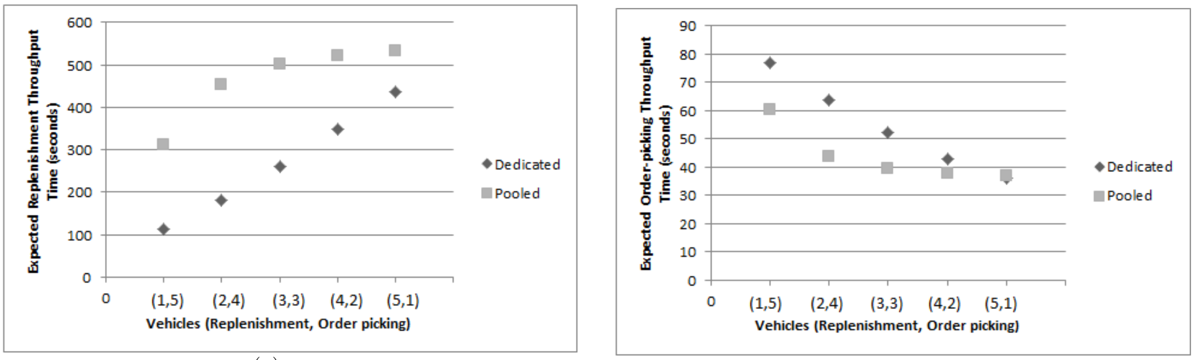
Figure 9: System throughput: random vs. closest open location storage policy

only $N - m + 1$ locations (and thus the aisle travel distance gets shorter), and therefore the throughput increases. Note that although open locations are not used efficiently in the closest open location storage policy, the used locations are more close to the aisle entrance which reduces the robot travel times.

6.2 Insights for the Multi-class Queueing Network Model

6.2.1 Dedicated class of robots for order picking and replenishment

Our observations on better system design with the dedicated robot classes are similar to the single-class case. For instance, from Table 3, we see that by increasing the locations in a warehouse, the waiting times at the aisles increase. This results in throughput reduction for both order picking and replenishment classes. Likewise, from Table 3 with 200 location, we observe that when the ratio of $\frac{D}{W}$ increases from 1 to 2, the robots take about 1-3% less throughput times and the system throughput increases from 1-3%. Similarly, with 400 locations, by increasing the $\frac{D}{W}$ ratio from 1 to 2, the robots take 2-3% less cycle times and results in 1-3% more throughput capacity. We observe that an increase in $\frac{D}{W}$ ratio has a marginal impact on the system performance measures.



(a) (b)

Figure 10: Comparison of expected replenishment and order picking throughput times (a and b) with dedicated and pooled robots

6.2.2 Pooled class of robots for order picking and replenishment

In this case, a robot can process both order picking and replenishment activity. From Table 4, it can be seen that the insights that hold for a single-class or a multi-class with dedicated robots also hold true for the pooled robot system.

This work is motivated to the answer the question if pooled robots in usage perform better than dedicated robots for order picking and replenishment activity.

From the numerical results, we see that dedicated robots take less throughput times than the pooled robots for the order picking process whereas the dedicated robots that more throughput times than the pooled robots for the replenishment process. In the pooled classes, we use a 80% chance that the robots can come switch to the order picking process and 20% chance that the robots switches to the replenishment process. Note that a pooled resource allocation may result in more variability in the queue lengths at the order pick and replenishment stations. Further, the replenishment station has a longer service time requirements than the order pick service time. Therefore, pooled robot assignment to the replenishment process results in two to three times more average waiting time than the dedicated robot assignment (refer Tables 3 and 4). However, the pooled robot assignment results in 30%-60% reduction in order pick throughput times in comparison to the dedicated robot assignment.

In Figure 10, expected replenishment and order pick throughput times are compared for a system with six robots but different robot allocations. For instance, if the number of robots dedicated to replenishment and order picking are 2 and 4 respectively, then we compare the average throughput time measures with a pooled system where the robot switches to replenishment and order picking with probability (2/6) and (4/6) respectively after completing every process cycle. We see that for all set of robot allocations, the average queue length at the order pick station is lower for the pooled case than the dedicated robot case. However, the average queue length at the replenishment station is higher for the pooled case than the dedicated case. Therefore the the overall waiting time at the order pick station decreases in the pooled robot case, which decreases the overall throughput time for order picking (upto 60% less).

7 Conclusion and Future Scope

In this paper, we present analytical models to estimate the performance measures of mobile-shelves based order pick systems. Using these models, we analyze system performance with alternative pod storage strategies and system operations with dedicated or pooled robots. We see that although the open locations are not efficiently used in the closest open location storage policy, the throughput time estimate in this policy is always less than the throughput time estimate obtained using the random open location storage policy. Our results show that the waiting times of robots are reduced in aisles with a closest open location storage policy and the throughput per hour with a closest open location storage policy is 1-3% higher than the random open location storage policy. Another important design parameter is the warehouse shape factor, $\frac{D}{W}$ ratio, which is defined as the ratio between length and width, which is represented by the number of aisles and number of rack locations, respectively, of the order picking system. We note that more congestion occurs in systems with smaller warehouse shape factors because of increase in waiting times within the aisles. We compared different design alternatives and recommend the system to be built with many short aisles instead of a few long aisles. Finally, we also observe that by using pooled robots instead of dedicated robots, the expected throughput time for order picking reduces upto one-third of its initial value; however, the expected replenishment time estimate increases upto three times.

We make multiple assumptions while developing the analytical models. Several of those assumptions, such as the probability of entering an aisle, storing the pod in the same side of the aisle from which the next pod needs to be retrieved, etc. can be relaxed in a straightforward fashion. Our model can also be extended to accommodate multiple pick and replenishment stations, zones, and realistic travel of robots underneath the pods.

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