A machine produces products in two phases. The first phase is standard and the same for all products. The second phase is customer specific (the finishing touch). The first (resp. second) phase takes an exponential time with mean $1/\mu_1$ (resp. $1/\mu_2$) hour. Orders for the production of one product arrive according to a Poisson stream with a rate of λ orders per hour. Orders are processed in order of arrival. The state of this system can be described by the pair (i, j) where i is the number of orders in the system and j indicates the production phase of the order that is being processed (so j is 1 or 2).

(i) Formulate and solve the balance equations for the equilibrium probabilities $p(i, j)$.

Suppose that $\mu_1 = 6, \mu_2 = 30$ and $\lambda = 3$.

(ii) Determine the mean production lead time (waiting time plus production time) of an order.

The machine is switched off when the system is empty and it is switched on again as soon as the first order arrives. A fixed cost of 20 dollar is incurred each time the machine is switched on (the time needed to switch the machine on or off is negligible).

(iii) Determine the average switch-on cost per hour.

To reduce the production lead time one decides to start already with the production of phase 1 when the system is empty. If upon completion of phase 1 no order has arrived yet, the production stops and the machine is switched off. When the first order arrives the machine is switched on again and can directly start with phase 2.

(iv) Determine the reduction in the mean production lead time.

(v) Determine the average switch-on cost per hour.