Consider a machine where jobs are being processed. The mean production time is 4 minutes and the standard deviation is 3 minutes. The mean number of jobs arriving per hour is 10. The interarrival times are uniformly distributed between 4 and 8 minutes.

(i) Show that, if the production time is represented as a mixture of an Erlang-1 (i.e. exponential) distribution and an Erlang-2 distribution with density

$$f(t) = p\mu e^{-\mu t} + (1-p)\mu^2 t e^{-\mu t},$$

the paramaters p and μ can be chosen such that this distribution fits the mean and standard deviation of the production time.

The state of this system just prior to the arrival of an order can be described by n, where n is the number of uncompleted production phases in the system.

(ii) Formulate and solve the balance equations for the arrival probabilities a_n.

(iii) Determine the mean waiting time of a job.

(iv) What is the fraction of jobs that has to wait longer than 10 minutes?