Consider a machine processing orders. These orders arrive according to a Poisson process with a rate of 1 order per hour. When there are no orders, the machine is switched off. As soon as a new order arrives, the machine is switched on again, which takes exactly T hours. The processing time of an order is exponentially distributed with a mean of 30 minutes.

(i) Determine as a function of T:

(a) The mean numbers of orders processed in a production cycle.

(b) The mean duration of a production cycle.

(c) The mean production lead time of an order.

Suppose that it costs 17 dollar each time the machine is switched on again and t hat the waiting cost per hour per order are 1 dollar.

(ii) Show that the average cost per hour are minimal for $T = 3$ hour.