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Department of Mathematics and Computer Science
Solutions of exercises Stochastic Processes 2 (2S480) for week 6, 2006.

1. The random variable N is equal to $N(1) + 1$ where $\{N(t, t \geq 0\}$ is the renewal process whose interarrival distribution is uniform on $(0, 1)$. By the result of Example 7.3,

$$E[N(t)] = m(1) + 1 = e.$$

2. Yes, p/μ.

3.

$$\frac{N(t)}{t} = \frac{1}{t} + \frac{\text{number of renewals in } (X_1, t]}{t}.$$

Since $X_1 < \infty$, Proposition 7.1 implies that, as $t \to \infty$,

$$\frac{\text{number of renewals in } (X_1, t]}{t} \to \frac{1}{\mu}.$$

4. Let X be the time between successive d-events. Conditioning on the time until the next event follwoing a d-event gives

$$E[X] = \int_0^d x\lambda e^{-\lambda x} dx + \int_d^\infty (x + E[X])\lambda e^{-\lambda x} dx = 1/\lambda + E[X]e^{-\lambda d}.$$

Therefore,

$$E[X] = \frac{1}{\lambda(1 - e^{-\lambda d})}.$$

a. $\frac{1}{E[X]} = \lambda(1 - e^{-\lambda d})$.

b. $1 - e^{-\lambda d}$.

5.

a. X_i is the amount of time he has to travel after his ith choice (we will assume that he keeps making choices even after becoming free). N is the number of choices he makes until becoming free.

b.

$$E[T] = E\left[\sum_1^N X_i\right] = E[N]E[X].$$

N is a geometric random variable with $p = 1/3$, so $E[N] = 3$, $E[X] = \frac{1}{3}(2+4+6) = 4$. Hence, $E[T] = 12$.

c.

$$E\left[\sum_1^N X_i | N = n\right] = (n-1)\frac{1}{2}(4+6) + 2 = 5n - 3,$$

since, given $N = n$, $X_1, \ldots, X_{n-1}$ are equally likely to be either 4 or 6, $X_n = 2$. Further, $E[\sum_1^n X_i] = 4n$.

d. From c,

$$E\left[\sum_1^N X_i\right] = E[5N - 3] = 15 - 3 = 12.$$