The design of robotic dairy barns using closed queueing networks

I. Halachmi [a,b], I.J.B.F. Adan [c], J. van der Wal [c]
J.A.P. Heesterbeek [d], P. van Beek [e],

[a] Institute of Agricultural and Environmental Engineering (IMAG-DLO), Wageningen, The Netherlands
[b] Department of Agricultural Engineering and Physics, Wageningen Agricultural University (WAU), Wageningen, The Netherlands
[c] Department of Mathematics and Computing Science, Eindhoven University of Technology (TUE), Eindhoven, The Netherlands
[d] Centre for Biometry Wageningen (CBW), Wageningen, The Netherlands
[e] Department of Mathematics, Wageningen Agricultural University, Wageningen, The Netherlands

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Abstract

In this paper we present a closed queueing network model for a robotic milking barn. We use an approximative mean-value algorithm to evaluate important performance criteria such as the number of cows waiting, their waiting time and the utilization of the facilities in the barn. It is shown how the results from the queueing network analysis can support the discussion about the optimal design of the robotic barn.

Keywords: design of dairy barns, milking robot, queueing network, approximate mean value analysis

1 Introduction

The most important recent development in the dairy industry is robotic milking. Dairy barns with milking robots are becoming more and more interesting from an economical point of view, cf. [2]. Today already over 200 robots have been on installed on commercial Dutch farms. As robotic milking barns (RMB) are expensive, it is important to develop models which make it possible to discuss the optimal layout of an RMB and the optimal capacities of the various facilities in the barn depending on the herd size before actually constructing it.

In an experimental farm in Duiven in the Netherlands the agricultural research center IMAG-DLO is investigating the behaviour of the cows in an RMB. Based on extensive measurements and observations, it was in [3] concluded that it is necessary to incorporate the stochastic behaviour of the cows in the design of an RMB. Another aspect, which makes this design complex, is the interaction between the facilities in the barn: increasing the capacity of bottleneck facilities will shift queues and alter the location of bottlenecks, possibly forcing the designer to increase the capacity elsewhere. The concept of the closed queueing network(CQN) seems to be very appropriate for modelling and analysing an RMB. It covers both the random behaviour of the cows and the interaction between the queues. It also supports a systematic analysis of the economic tradeoffs. The CQN model is widely used in the communication systems and production systems areas. The present application area, the design of RMBs, is new. As we will see, it is a potentially powerful design tool here as well.

In section 2 we briefly describe the milking robot. In section 3 we look at the RMB and we introduce the CQN model. We discuss the data that are needed and we already formulate a number of performance criteria that are particularly important in this application. In section 4 we present the approximate mean-value analysis of the CQN. This approximation technique is validated in section 5 by comparing it with simulation of the CQN model. The simulation uses real data collected in the experimental barn in Duiven. In section 6 we show how the results from the CQN analysis support the discussion about the possible designs of the barn. In section 7 we spend a few lines on the Java applet that has been build for this application. The final section is devoted to conclusions and comments.

2 The milking robot

The milking robot is shown in figure 1. Milking robots are different from the ordinary milking machines in one crucial aspect: the robot uses sensors to find the teats of the cow and then connects the cups to the teats with a robot arm.

There are at least two good reasons for using robots. First, it saves a serious amount of labour and second, it makes it possible to go from milking twice a day to three or even more times a day. When cows are milked three times a day their production is increased by about 15 percent. Milking robots, their operation and costs have been reviewed elsewhere, see e.g. [1, 11, 10].

3 The RMB and the CQN

In this section we describe the RMB, formulate the CQN model and discuss some of the performance aspects.

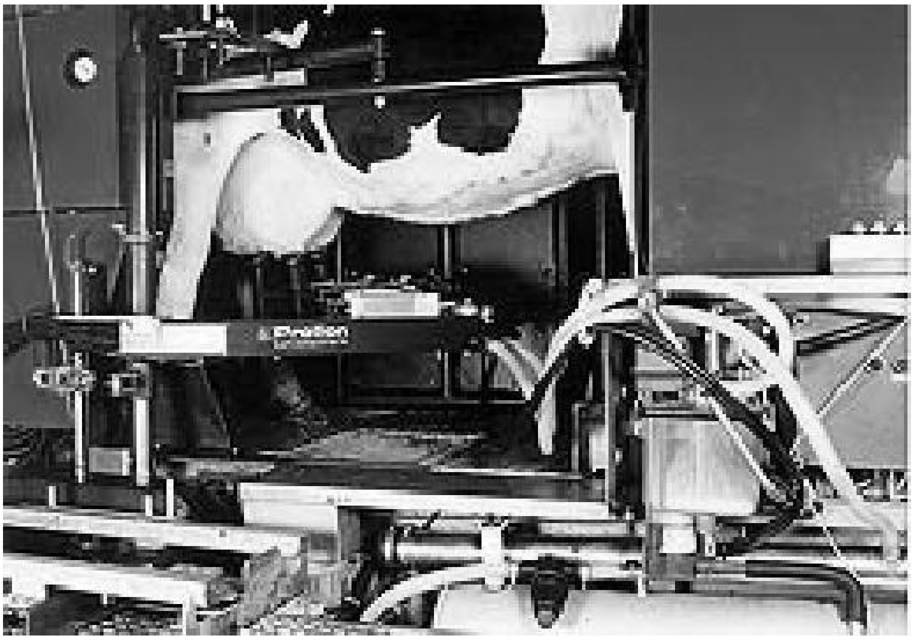

Figure 1: A milking robot (source: IMAG-DLO)

3.1 The RMB

The basic layout of the RMB we are dealing with is shown in figure 2. In the barn we distinguish five facilities or servers.

- The Milking robot.
- The second one is the Concentrate feeder. Each cow is allowed to receive only a limited amount of 'concentrate', cf. [6]. So the Concentrate feeder must have the equipment to be able to identify the cows and to decide how much concentrate to give to the cow. Cows are very fond of the concentrate. Therefore the Concentrate feeder can be and is used to get the cows to pass through the Milking robot. In the present design in Duiven the cows can only reach the Concentrate feeder via the robot.

The three more conventional facilities are:

- The Forage lane. Forage lanes are cheap. There are no limitations on foraging. The only condition is that there must be enough eating positions at the forage feeder to prevent the cows from becoming aggressive.
- The Water troughs. A 'high-yielding' cow may drink upto 180 liters a day. Water throughs are cheap, but of significant physiological importance.

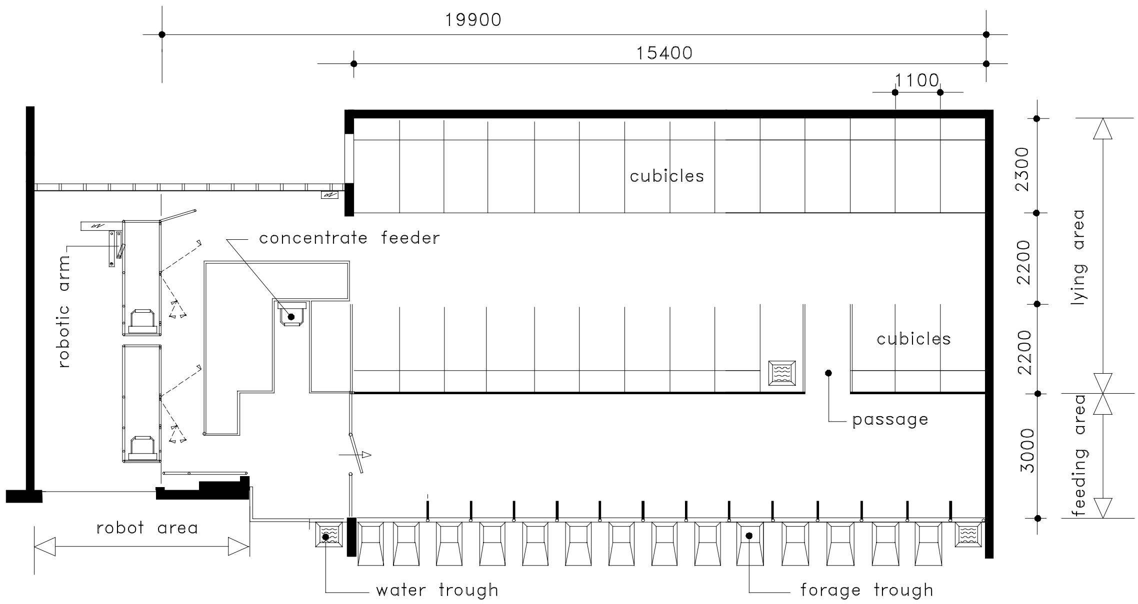

Figure 2: Layout of the experimental barn in Duiven. Dimensions in mm. (source: IMAG-DLO)

- The Cubicles. In the cubicles the cows can lay down, rest, and avoid confrontations. They spend roughly 50 percent of their time in the cubicles. Cubicles only require space, some fencing and bedding material (wood shavings, sand, rubber mattresses).

Further we need one more, artificial, facility that we will call:

- Walking. The space in between the facilities is used for walking, idling or grouping. This takes nearly 25 percent of their time, so 5 to 6 hours a day. In that time they cover at most a few kilometers, so a better word for the facility might be 'Standing.' Anyway, the walking area should be large enough to accommodate somewhat more than 25 percent of the herd.

3.2 The CQN model

The above description suggests a CQN model with 6 stations:

1. Milking robot,
2. Concentrate feeder,
3. Forage lane,
4. Water trough,
5. Cubicle and
6. Walking.

Walking is modelled as an infinite server. The other stations are single of multiple servers stations. In this CQN the customers are the cows. The number of cows is fixed and denoted by H (for herd).

3.3 The service times and visit frequencies

For the CQN model we need for each station the service time and the relative number of visits. This data is obtained from measurements in the experimental barn in Duiven. An extensive presentation and discussion of the measurements can be found in [3]. We note, however, that the visit frequencies depend also on the layout. In Duiven part of the visits of the cows to the facilities Milking robot and Concentrate are not successful. The milking frequency is limited to once every 6 hours and they only receive concentrate after being milked. Figure 3 gives the frequency distribution of the service time in the Milking robot obtained from the measurements. The small service times, but also some of the very long service times correspond to unsuccessful visits of a cow to the robot. In the approximative analysis of the CQN model, however, we do not use the complete distribution of the service time. We only need its mean and standard deviation. The data we use is displayed in table 1. The sum of the visit frequencies is normalised to 1.

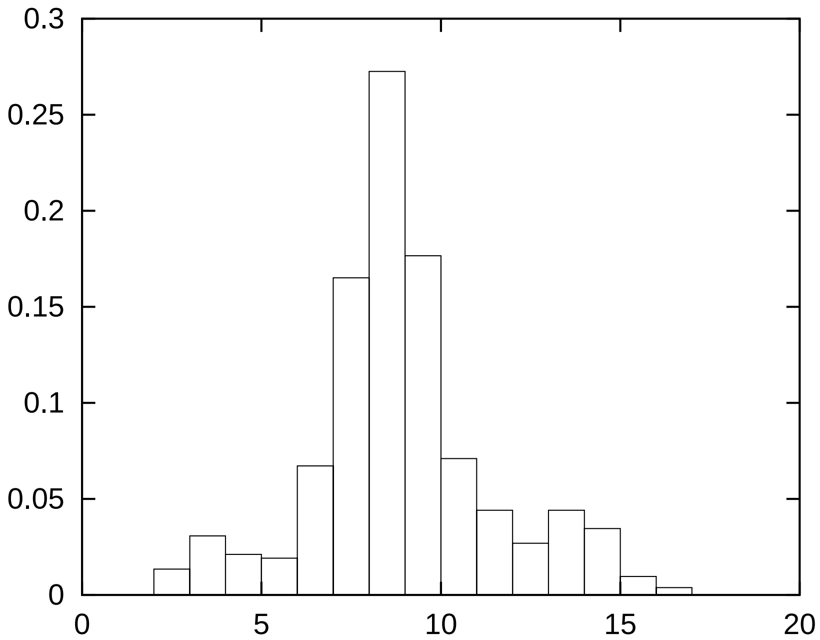

Figure 3: Frequency distribution of the service time (in min.) in the Milking robot

Walking is modelled as an infinite server, so there is no waiting for Walking. Therefore it does not make any difference whether the time a cow spends in Walking consists of many short periods or a few longer ones. In the model we use this freedom to set the visit frequency for Walking equal to 1. From the measurements in Duiven we know that Walking

Facility	Relative visit frequency	Service time (in min.) Mean	Standard deviation
Milking robot	0.164	8.41	2.52
Concentrate feeder	0.155	6.38	6.25
Forage lane	0.235	15.0	11.9
Water trough	0.170	3.18	2.30
Cubicle	0.276	38.9	60.3

Table 1: Service requirements in the facilities in the barn

takes 23.8 percent of the time.[1] So one 'visit' to Walking should take 23.8/(100−23.8) of the mean time needed for one visit to the other facilities (weighted with the visit frequencies). Since the herd in the barn in Duiven is small compared to the various capacities, cows rarely have to wait for service. So the mean time needed for one visit is equal to the mean service time. Hence, from the data in table 1 we obtain a mean walking time per visit of 5.36 minutes. In table 2 we show how according to the measurements a cow spends the time in the various facilities.

Facility	Percentage of time
Milking robot	6.1
Concentrate feeder	4.4
Forage lane	15.7
Water trough	2.4
Cubicle	47.7
Walking	23.8

Table 2: Percentages of time a cow spends in the various facilities

3.4 The performance and design criteria

In a CQN we are normally interested in the mean waiting times and the means or distributions of the numbers of 'customers' or 'jobs' at the various stations. In our case that is not very different. Queueing is something cows do not like. In this respect they do not differ from humans. When a cow is waiting for a facility and another cow arrives, aggressive behaviour might occur. Particularly at the scarce facilities Milking robot and Concentrate waiting has to be limited. Some waiting for Cubicles is not really a problem because Walking seems to be an alternative for the Cubicles. So in the discussion about the design we will focus on mean waiting times and the queue lengths at the stations Milking robot and

[1]The numbers determined from the data in Duiven are given in 3 digits. Of course, the measurements do not guarantee this accuracy.

Concentrate feeder and on the maximum number of cows that can be accommodated in a certain design.

Given the visit frequencies, service times, capacities and herd size we can evaluate waiting times, queue lengths and utilizations for the various facilities. From these we can discuss and judge the design under consideration.

In the next section we present the algorithm for evaluating the performance.

4 The AMVA

A standard technique for the analysis of closed queueing networks is mean-value analysis (MVA) (see, e.g. [7, 8]). The name MVA refers to the fact that it deals with relations between the quantities:

- mean time spent in a station,
- mean number of cows in a station, and
- mean number of visits per time unit to a station.

Exact MVA is based on Little's formula [5] and the arrival theorem [4] stating that in a CQN a customer moving from one station to another sees the network in equilibrium with one customer removed. Little's formula is valid under general circumstances, but the arrival theorem only holds exactly for product-form networks. The CQN model of the barn has no product-form solution, but we will adopt the arrival theorem as an approximation. We will denote this approximate MVA by AMVA.

Before formulating the AMVA relations we introduce some notation. The stations are numbered according to the list in subsection 3.2. The relative visit frequency to station i is v_i and the mean and standard deviation of the service time in station i are denoted by s_i and σ_i, respectively. The mean residual service time R_i in station i upon an arrival instant is approximated by (see [9])

$$R_i = \frac{s_i}{2}\left(1 + \left(\frac{\sigma_i}{s_i}\right)^2\right).$$

The number of servers in station i is c_i. We further introduce the following quantities depending on the herd size H,

$W_i(H)$	mean waiting time in station i,
$S_i(H)$	mean visit time in station i (waiting plus service time),
$\Lambda_i(H)$	mean number of visits per time unit to station i,
$L_i(H)$	mean number of cows waiting (not in service) in station i,
$\rho_i(H)$	server utilization in station i,
$Q_i(H)$	probability that all servers are busy in station i.

We now formulate the relations for $W_i(H)$, $S_i(H)$, $\Lambda_i(H)$ and $L_i(H)$. The relation for $W_i(H)$ depends on whether station i is a single, multi or infinite server. In the infinite server station Walking (station 6) there is no waiting, so

$$W_6(H) = 0.$$

The other stations are single or multi server. In a single-server station we use

$$W_i(H) = \rho_i(H-1)R_i + L_i(H-1)s_i, \tag{1}$$

where

$$\rho_i(H-1) = \Lambda_i(H-1)s_i.$$

In a multi-server station the relation for the mean waiting time is a bit more complicated. If not all servers are busy the waiting time is zero. If all servers are busy and there are 0 or more jobs waiting then the new arrival first has to wait until the first departure and then it continues to wait for as many departures as there were jobs waiting upon arrival. As an approximation we assume that with c servers the time till the first departure and the time to clear the queue is c times smaller than with one server. So we use the approximation,

$$W_i(H) = Q_i(H-1)\frac{R_i}{c_i} + L_i(H-1)\frac{s_i}{c_i}. \tag{2}$$

Clearly, for $H < c_i$ the probability $Q_i(H)$ is equal to 0. Otherwise it is approximated by the corresponding probability in an $M/M/c_i$ system (see, e.g. [12]) with arrival rate $\Lambda_i(H)$ and mean service time s_i. So, for $H \geq c_i$, writing $\rho_i(H) = \Lambda_i(H)s_i/c_i$,

$$Q_i(H) = \frac{(c_i\rho_i(H))^{c_i}}{c_i!}\left\{(1-\rho_i(H))\sum_{k=0}^{c_i-1}\frac{(c_i\rho_i(H))^k}{k!} + \frac{(c_i\rho_i(H))^{c_i}}{c_i!}\right\}^{-1}.$$

Note that for $c_i = 1$ equation (2) reduces to the single server equation (1), and that for $c_i = \infty$ it simplifies to $W_i(H) = 0$. Equations (1) and (2) express the mean waiting time for herd size H in terms of quantities for herd size $H-1$. So they are recursive in H. We emphasize that the equations (1) and (2) for the mean waiting time are approximations, and that, of course, also other approximate equations are possible.

The mean visit time in station i is the sum of the mean waiting time and the mean service time, so

$$S_i(H) = W_i(H) + s_i\,.$$

To determine the arrival rates, i.e., the mean number of visits per time unit, $\Lambda_i(H)$ we first note that the mean time elapsing between the starts of two successive walks, $C(H)$ say, is equal to (recall that $v_6 = 1$)

$$C(H) = \sum_{i=1}^{6} v_i S_i(H)\,,$$

since inbetween two walks a cow visits facility i on the average v_i times. Hence, the mean number of visits per time unit to facility i of one cow is $v_i/C(H)$. Since there are H cows around, we have

$$\Lambda_i(H) = \frac{v_i H}{C(H)} = \frac{v_i H}{\sum_{i=1}^{6} v_i S_i(H)}.$$

Finally, Little's formula applied to the queue in station i yields

$$L_i(H) = \Lambda_i(H) W_i(H)\,.$$

This completes the set of AMVA relations. The relations can be solved recursively. Starting with an empty barn for which $L_i(0) = \Lambda_i(0) = 0$, we can subsequently compute the quantities $W_i(h)$, $S_i(h)$, $\Lambda_i(h)$, $Q_i(h)$ and $L_i(h)$ for $h = 1, \ldots, H$ using the relations formulated above. The AMVA algorithm is summarized in figure 4.

Step 1. *Initialisation:* $L_i(0) = Q_i(0) = 0$ *for all* i.

Step 2. *For* $h = 1, 2, ..., H$ *compute for* $i = 1, \ldots, 6$

$$\begin{aligned}
W_i(h) &= Q_i(h-1)\frac{R_i}{c_i} + L_i(h-1)\frac{s_i}{c_i}, \\
S_i(h) &= W_i(h) + s_i, \\
\Lambda_i(h) &= \frac{v_i h}{\sum_{i=1}^{6} v_i S_i(h)}, \\
L_i(h) &= \Lambda_i(h) W_i(h), \\
\rho_i(h) &= \Lambda_i(h)\frac{s_i}{c_i}, \\
Q_i(h) &= 0, \qquad \textit{if } h < c_i, \\
&= \frac{(c_i\rho_i(h))^{c_i}}{c_i!}\left\{(1-\rho_i(h))\sum_{k=0}^{c_i-1}\frac{(c_i\rho_i(h))^k}{k!} + \frac{(c_i\rho_i(h))^{c_i}}{c_i!}\right\}^{-1}, \ \textit{if } h \geq c_i.
\end{aligned}$$

Figure 4: AMVA algorithm

Based on $Q_i(H)$ and $L_i(H)$ we can compute an approximation for the queue length probabilities. Let $p_i(k, H)$ denote the probability of k cows (waiting or in service) in station i. For $k > c_i$ we set (as an approximation)

$$p_i(k, H) = (H - k + 1)\alpha p_i(k-1, H) = \cdots = (H - c_i)_k \alpha^{k-c_i} p_i(c_i, H), \tag{3}$$

where $(n)_k = n(n-1)\cdots(n-k+1)$. The factor $H - k + 1$ reflects the assumption that the arrival rate will be proportional to the number of cows that is not in station i. The two unknowns α and $p_i(c_i, H)$ are determined such that

$$\sum_{k=c_i}^{H} p_i(k,H) = Q_i(H), \qquad \sum_{k=c_i}^{H} (k-c_i)p_i(k,H) = L_i(H).$$

In the next section we will investigate the accuracy of AMVA.

5 Validation

We should verify that the CQN model is a reasonable representation of reality, and that AMVA produces good approximations for the performance of the CQN model.

A real life validation of the CQN model is complicated. The cows need a serious amount of time to adjust to a new layout and changes in the herd. Further, many other parameters, e.g., climate conditions or an occasional illness, influence the accuracy of the measurements. But we can say that the CQN model exactly describes the relative workloads of the facilities in the barn, and it takes into account the variability in the service times and the interference effects between the facilities. Further the performance of the CQN model has repeatedly passed the 'face validation' test as several people familiar with the barn district found it accurately mimicking a real system.

Scenario			Milking robot						Concentrate feeder					
			$\rho_1(H)$		$W_1(H)$		$L_1(H)$		$\rho_2(H)$		$W_2(H)$		$L_2(H)$	
c_1	c_2	H	amva	sim	amva	sim	amva	sim	amva	sim	amva	sim	amva	sim
1	1	10	.580	.578	4.26	4.41	.294	.303	.416	.418	3.48	2.81	.227	.184
2	1	10	.298	.296	.332	.344	.024	.024	.427	.428	3.58	3.50	.240	.235
		15	.434	.431	.833	.868	.086	.089	.623	.624	7.47	7.26	.729	.711
		20	.551	.548	1.60	1.65	.209	.216	.791	.793	14.4	14.0	1.78	1.74
2	2	20	.599	.595	1.89	1.97	.269	.278	.430	.431	1.17	.941	.157	.127
		25	.736	.731	3.60	3.83	.631	.666	.528	.530	1.99	1.54	.329	.256
		30	.858	.852	6.67	7.41	1.36	1.50	.615	.617	3.12	2.28	.601	.440
3	2	30	.594	.590	1.03	1.07	.217	.226	.639	.640	3.39	3.16	.678	.634
		35	.680	.677	1.69	1.76	.411	.426	.732	.734	5.35	4.95	1.23	1.14
		40	.757	.754	2.66	2.81	.719	.755	.814	.819	8.29	7.70	2.12	1.98
3	3	40	.789	.785	3.06	3.26	.861	.913	.566	.568	1.27	.956	.339	.256
		45	.868	.866	5.00	5.59	1.55	1.73	.623	.626	1.81	1.30	.530	.384
4	3	45	.668	.667	1.04	1.12	.331	.354	.638	.644	1.94	1.77	.583	.536
		50	.724	.726	1.54	1.73	.529	.599	.692	.701	2.71	2.55	.883	.841

Table 3: Comparison of AMVA with simulation results

To verify the accuracy of the AMVA we compare it with simulation of the CQN. The examples are based on the real barn measurements in Duiven. AMVA only needs the data in table 1, but the simulation uses more detailed information, i.e., the transition probabilities between the facilities and the frequency distributions of the service times. The walking times in the simulation model are taken to be exponential. In table 3 we list the performance of the Robot and Concentrate feeder for several scenarios. In each of the scenarios we have 12 eating positions at the forage lane, 3 water troughs and 27 cubicles. The accuracy of the simulation results is 0.1–0.5% for the utilization and 1–2% for the mean waiting time and mean queue length. The simulation time on a SUN-5 170Mhz Workstation is for each scenario approximately 8 minutes. The computation time for AMVA is negligible.

The results in table 3 show that AMVA predicts the utilizations and hence the arrival rates perfectly. The predictions for the mean waiting times and mean queue lengths are good. For design purposes, we may conclude that AMVA is sufficiently accurate.

In table 4 we consider the approximation for the queue length probabilities, and compare it with simulation. We list $P_i(k, H)$, which is the probability that in station i all servers are busy and k or more cows are waiting for service.

Scenario				$P_1(k,H)$				$P_2(k,H)$			
c_1	c_2	H	k	0	1	2	3	0	1	2	3
1	1	10	amva	.580	.206	.065	.018	.416	.155	.052	.015
			sim	.577	.223	.063	.014	.418	.135	.037	.009
2	1	10	amva	.137	.021	.003	.000	.427	.163	.055	.016
			sim	.124	.021	.002	.000	.428	.161	.053	.015
2	1	15	amva	.263	.066	.015	.003	.623	.361	.195	.098
			sim	.246	.070	.015	.003	.624	.358	.189	.093
2	1	20	amva	.392	.140	.047	.015	.791	.597	.431	.297
			sim	.373	.149	.049	.014	.793	.596	.424	.289
2	2	20	amva	.449	.174	.064	.022	.258	.101	.037	.013
			sim	.427	.185	.066	.020	.246	.088	.028	.008
2	2	25	amva	.624	.327	.164	.079	.364	.179	.084	.038
			sim	.602	.352	.179	.081	.350	.155	.063	.024
2	2	30	amva	.792	.528	.340	.211	.469	.274	.155	.085
			sim	.774	.576	.391	.245	.454	.237	.114	.052
3	2	30	amva	.347	.136	.052	.019	.498	.300	.174	.098
			sim	.320	.144	.055	.019	.493	.293	.166	.091
3	2	35	amva	.464	.224	.105	.047	.618	.432	.294	.194
			sim	.436	.235	.111	.048	.615	.424	.281	.180
3	2	40	amva	.578	.331	.184	.100	.730	.575	.443	.333
			sim	.553	.350	.201	.106	.730	.567	.427	.313
3	3	40	amva	.630	.376	.219	.124	.312	.167	.087	.044
			sim	.600	.400	.241	.135	.291	.141	.064	.028
3	3	45	amva	.762	.532	.364	.243	.384	.230	.134	.076
			sim	.741	.575	.417	.286	.364	.196	.100	.048
4	3	45	amva	.380	.180	.084	.038	.405	.247	.147	.085
			sim	.353	.192	.092	.040	.400	.238	.137	.076
4	3	50	amva	.467	.254	.135	.070	.481	.322	.212	.136
			sim	.453	.282	.161	.084	.484	.320	.206	.128

Table 4: Queue length probabilities for the Robot and Concentrate feeder

The results show that AMVA produces good approximations. Clearly, the inaccuracy of the input data is more important than the inaccuracy of AMVA. In the next section we demonstrate how AMVA can be used as a practical design tool.

6 Applications

A practical problem related to the design of a barn is for example the following. A farmer considering to buy additional milk quota wants to know whether the present capacity of the facilities is sufficient for holding a bigger herd. And if not, how much extra capacity is needed. Below we show how AMVA can be used in this situation.

We consider the barn described in section 3. The initial configuration is 1 milking robot, 1 concentrate feeder, 12 forage lane eating positions, 3 water troughs and 27 cubicles. The design criterion is an upper limit of 2 minutes for the mean waiting time in each facility. In table 5 we list for various herd sizes the minimal number of servers needed in each facility. This number is determined by using the following add-heuristic. We start with the initial configuration. If the mean waiting time in each facility is less than 2 minutes, we are done. Otherwise, we add one server to the facility with the greatest mean waiting time (i.e. the bottle-neck facility) and we repeat this procedure until the mean waiting time in each facility drops below the upper limit of 2 minutes.

						Robot		Concentrate		Cubicles	
H	c_1	c_2	c_3	c_4	c_5	$\rho_1(H)$	$W_1(H)$	$\rho_2(H)$	$W_2(H)$	$\rho_5(H)$	$W_5(H)$
10	2	2	12	3	27	.305	.345	.219	.240	.176	.000
20	2	2	12	3	27	.599	1.89	.430	1.17	.345	.000
30	3	3	12	3	27	.605	1.08	.434	.531	.523	.003
40	4	3	12	3	27	.602	.668	.575	1.33	.694	.191
50	4	4	12	3	28	.738	1.64	.529	.596	.821	1.43
60	5	4	12	3	32	.701	.957	.628	1.19	.852	1.89

Table 5: Minimal number of servers such that $W_i(H) \leq 2$ minutes for all i

Table 5 shows that many expensive robots are needed to keep the waiting times small. The reason is inefficient use of the robots. In the present layout each cow visits the robot nearly 10 times per day. A cow is milked 3 times a day. So the other 7 times, the cow occupies the robot, not because she has to be milked, but because she wants concentrate. Unsuccessful visits to the robot may be avoided by means of a selective gate in front of the robot, through which only cows may pass who have to milked. The effect of a selective gate on the required milking capacity can be evaluated with AMVA. It is not completely clear how this will alter the service times of the robot. We will assume that the service time of a successful visit to the robot has the same mean and standard deviation as in table 1. This is reasonable for its mean, but probably its standard deviation will be smaller (cf. subsection 3.3). We only have to reduce the visit frequencies to the Robot and the Concentrate feeder (since it can be reached only by passing through the Robot, see figure 2) with nearly 70 percent to 0.05 (see table 1). In table 6 we list for various herd sizes the minimal number of servers needed in each facility. The result is a substantial cost saving:

for a herd of, e.g., 50 cows we now need 2 instead of 4 robots, and 2 instead of 4 feeders. On the other hand, some extra cubicles are required, but they are not expensive.

						Robot		Concentrate		Cubicles	
H	c_1	c_2	c_3	c_4	c_5	$\rho_1(H)$	$W_1(H)$	$\rho_2(H)$	$W_2(H)$	$\rho_5(H)$	$W_5(H)$
10	1	1	12	3	27	.200	.980	.152	.971	.189	.000
20	2	2	12	3	27	.201	.169	.152	.132	.380	.000
30	2	2	12	3	27	.301	.412	.228	.314	.569	.011
40	2	2	12	3	27	.398	.788	.302	.583	.752	.487
50	2	2	12	3	30	.489	1.32	.371	.938	.832	1.40
60	3	2	12	3	35	.387	.305	.440	1.42	.846	1.42

Table 6: Minimal number of servers such that $W_i(H) \leq 2$ minutes for all i in a barn with a selective gate

7 The Java applet Cow

The CQN model of the robotic barn has been implemented in a user-friendly Java applet called *Cow*. The performance of the barn can be easily evaluated with the applet. It offers several possibilities to show the results, i.e. in the form of bar charts, pie charts or simple text charts. The applet Cow can be used freely on the World Wide Web. The URL is: `http://www.win.tue.nl/math/bs/stoch_opt/queueing_applets/cow.html`

8 Conclusion

In this paper we presented a CQN model for a robotic barn. Since the CQN cannot be solved exactly, we used an AMVA. The computation time on a PC of AMVA is negligible and it produces good approximations. The CQN model provides a practical tool to support the design of robotic barns.

QN techniques are still uncommon in the analysis and design of livestock housing. As we demonstrated in the present study, these techniques appear to be very useful for design problems in this area as well.

References

[1] S. Devir, *The dairy control and management system in the robotic milking farm.* Ph.D. thesis, Wageningen Agricultural University, Wageningen, The Netherlands (1995).

[2] A.A. Dijkhuizen, R.B.M. Huirne, S.B. Harsh and R.W. Gardner, *Economics of robot application.* Computers and Electronics in Agriculture, 17 (1997), pp. 111–121.

[3] I. Halachmi, J.H.M. Metz, E. Maltz, A.A. Dijkhuizen and L. Speelman, *Designing the optimal robotic barn, Part 1: facility usage.* Submitted to Journal of Agricultural Engineering Research (1998).

[4] S.S. Lavenberg and M. Reiser, *Stationary state probabilities of arrival instants for closed queueing networks with multiple types of customers.* J. Appl. Probability, 17 (1980), pp. 1048–1061.

[5] J.D. Little, *A proof of the queueing formula* $L = \lambda W$. Opns. Res., 9 (1961), pp. 383–387.

[6] N. Livshin, E. Maltz and Y. Edan, *Regularity of dairy cow feeding behaviour with computer-controlled feeders.* Journal of Dairy Science, 78 (1995), pp. 296–304.

[7] M. Reiser and S.S. Lavenberg, *Mean value analysis of closed multichain queueing networks.* J. Assoc. Comput. Mach., 27 (1980), pp. 313–322.

[8] M. Reiser, *Mean-value analysis and convolution method for queue-dependent servers in closed queueing networks.* Performance Evaluation, 1 (1981), pp. 7–18.

[9] S.M. Ross, *Introduction to Probability Models.* 6th ed., Academic Press, London, 1997.

[10] W. Rossing, E. Aurik and W. Smit, *Robot milking systems and the integration in the dairy farm.* In: Fourth international dairy housing conference, 1998, St. Louis, Missouri, ed. J.P. Chastain, published by ASAE, pp. 61–70.

[11] B.R. Sonck, *Labour organisation on robotic milking dairy farms.* Ph.D. thesis, Wageningen Agricultural University, Wageningen, The Netherlands (1996).

[12] H.C. Tijms, *Stochastic Modelling and Analysis: A Computational Approach.* Wiley, Chichester, 1986.