New applications of the h-diameter

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Project summary

Let D denote the diameter of a graph G. Chung, Delorme and Solé [2] introduced the h-diameter of a graph as a natural extension of D. The *h-diameter of a graph G*, denoted by $D_h(G)$, is defined as the largest pairwise minimum distance of a set of h vertices in G. Note that for $h = 2$ this is equivalent to the regular definition of the diameter of the graph. While the diameter of a graph measures how far two distinct points can be, the h-diameter measures how far h points can be.

While there exist polynomial time algorithms to calculate the diameter of a graph [1], the complexity of the h-diameter is NP-hard [4].
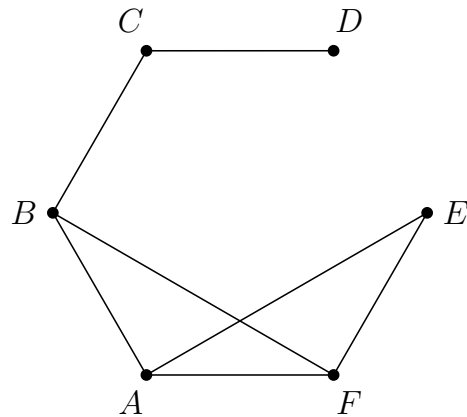

Figure 1: Illustration of the definition of the h-diameter. The graph has a 2-diameter of 4, given by vertex set $\{D, E\}$, as no other pair of vertices exists at a larger distance. Similarly, the 3-diameter is 2, determined by $\{B, D, E\}$, and for $h = 4, 5, 6$ the diameter is given by 1, as no set of four vertices exists with a pairwise distance greater than 1.

Since its introduction, the h-diameter has been extensively studied from the graph theoretical point of view. This thesis aims to explore new connections of the h-diameter in the field of combinatorial optimization. In particular, we aim at stydying the h-diameter of polytopes. Indeed, the concept of diameter of polytopes plays a crucial role in discrete mathematics (see for instance the survey [3]), but the h-diameter does not seem to have been deeply investigated so far for $h > 1$.

References

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[3] F. Santos. Recent progress on the combinatorial diameter of polyhedra and simplicial complexes. *Symposium on Computational Geometry* (2013) 165-166.

[4] H. Koerts. On the k-Independent Set Problem. *Bachelor thesis TU/e*, 2021.