Study Guide 2DMM10, updated Friday 22$^{\text{nd}}$ November, 2019

Supervised learning (BZ = "begeleide zelfstudie"):

Problems for supervised learning (BZ) are are in Dutch (old ones) or English (more recent ones), and are taken from bmia.bmt.tue.nl/people/lflorack/ (follow the link 2DMM10 & 8D010). If possible, please ask a fellow student to help you in case of Dutch language problems.

Oral lectures (C = "college"):

The course notes used for oral lectures (C) are in English and can be found on this webpage as well. Please consult the latest version 5.0 and check for updates indicated by the date stamp on the cover page (minor modifications will not affect the version number).

Examination:

You may use your own *printed* version of the course notes for the open book exam, provided it is in immaculate state. Hand-written annotations or minor remarks, such as corrections, are permitted. Hardcopies used during the exam will be checked and confiscated if in violation of these rules. *No substitute will be provided!* Definitions and notations in the exam take precedence over those in the course notes in case of conflict. Other material is not permitted during the exam. All electronic equipment must be switched off.

News and updates:

These instructions have been updated on **Friday 22$^{\text{nd}}$ November, 2019, 11:28**.

Please consult this study guide (paying attention to the date stamp above) and other items posted in Canvas regularly for news and updates. Confirmed items in the study guide are marked with ✔, unconfirmed ones with ✗ (these will be confirmed later, but might be rescheduled, depending on course progress). You may also receive mailings about this course. Check regularly for the course code string "2DMM10" in your subject headers.

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Tentative Schedule C & BZ

✔ **C-1a:**

- Preliminaries, pp. 1–7, except the $*$-topic (“Condensed Indices”).

✔ **C-1b:**

- Self-study: Chapter 1, Sections 1.1–1.4, pp. 8–16.
- Chapter 2, Sections 2.1 & 2.2.

✔ **BZ-1:**

- Problem 1, homework assignment deadline December 22, 2004 (elementary calculus).
- Problem 2, homework assignment deadline December 22, 2004 (elementary calculus).
- Problem 1abc, homework assignment deadline March 8, 2006 (elementary calculus).
- Problem 1, homework assignment deadline November 22, 2006 (group).
- Problem 2, homework assignment deadline November 22, 2006 (group).
- Problem 3, exam June 28, 2006 (group).
- Problem 1ab, exam January 17, 2007 (group).
- Additional exercise:
  - Expand the following multi-index expression for the 2^{nd}-order Taylor polynomial in dimension $n=2$:
  $$f(\mathrm{x}) = \sum_{0\leq|\alpha|\leq 2} \frac{1}{\alpha!} \nabla_\alpha f(0) x^\alpha \,,$$
  in which $\mathrm{x} = (x, y)$, $0 = (0, 0)$, and $\alpha = (a, b)$.
  - Expand the following tensor-index expression for the 2^{nd}-order Taylor polynomial in dimension $n=2$:
  $$f(\mathrm{x}) = \sum_{0\leq k\leq 2} \frac{1}{k!} \sum_{i_1=1}^{n=2} \ldots \sum_{i_k=1}^{n=2} \frac{\partial^k f(0)}{\partial x^{i_1} \ldots \partial x^{i_k}} x^{i_1} \ldots x^{i_k} \,,$$
  in which $\mathrm{x} = (x^1, x^2) = (x, y)$, and $0 = (0, 0)$.
  - Confirm that both yield identical results and discuss notational differences.
  - In the *Einstein summation convention* the $\sum_{i_j}$-symbols in the tensor-index expression are suppressed:
  $$f(\mathrm{x}) = \sum_{0\leq k\leq 2} \frac{1}{k!} f_{i_1\ldots i_k} x^{i_1} \ldots x^{i_k} \qquad \text{in which } f_{i_1\ldots i_k} \stackrel{\text{def}}{=} \frac{\partial^k f(0)}{\partial x^{i_1} \ldots \partial x^{i_k}} \,.$$

- Solution:

$$f(x,y) = f(0,0) + \frac{\partial f(0,0)}{\partial x} x + \frac{\partial f(0,0)}{\partial y} y + \frac{1}{2}\frac{\partial^2 f(0,0)}{\partial x^2} x^2 + \frac{\partial^2 f(0,0)}{\partial x \partial y} xy + \frac{1}{2}\frac{\partial^2 f(0,0)}{\partial y^2} y^2 .$$

The successive terms arise from the choices $(a,b) = (0,0)$; $(a,b) = (1,0), (0,1)$; $(a,b) = (2,0), (1,1), (0,2)$ in multi-index notation, resp. $k = 0$; $k = 1, i_1 = 1, 2$; $k = 2, i_1 = 1, 2, i_2 = 1, 2$ in tensor-index notation. In the latter case identical terms arise as a result of commutativity of multiplication and differentiation:

$$\frac{1}{2}\frac{\partial^2 f(0,0)}{\partial x \partial y} xy + \frac{1}{2}\frac{\partial^2 f(0,0)}{\partial y \partial x} yx = \frac{\partial^2 f(0,0)}{\partial x \partial y} xy$$

✔ **C-2a:**

- Chapter 2, Section 2.2.

✔ **C-2b:**

- Chapter 2, Sections 2.2 & 2.3.

✔ **BZ-2:**

- Finish: BZ-1.
- Problem 2ab(cd), homework assignment deadline December 13, 2006 (linear space).
- Problem 2a, exam July 8, 2004 (linear space).
- Problem 1ab, exam June 14, 2005 (linear space).
- Problem 2a, exam April 29, 2008 (linear space).
- Problem 3, exam June 13, 2008 (linear space).
- Problem 1, exam January 17, 2011 (linear space).

✗ **C-3a:**

✗ **C-3b:**

✔ **BZ-3:**

- Finish: BZ-2.
- Problem 1, exam March 26, 2004 (norm).
- Problem 2, exam July 8, 2004.
- Problem 1, exam March 17, 2006 (norm).
- Problem 1, exam June 28, 2006 (inner product, norm).
- Problem 2, exam June 28, 2006 (inner product).

- Problem 2, homework assignment deadline January 11, 2006 (linear space, inner product).
- Problem 1, homework assignment deadline February 10, 2004 (linear space, linear operator).
- Problem 1, homework assignment deadline December 13, 2006 (linear space, linear operator).
- Problem 2, exam March 17, 2006 (seminorm).
- Problem 2, exam July 11, 2003 (linear space, distance).
- Problem 1, exam March 8, 2005 (linear space, algebra).
- Problem 1, exam June 14, 2005 (algebra).
- Problem 2, exam March 21, 2007 (algebra).

✗ **C-4a:**

✗ **C-4b:**

✔ **BZ-4:**

- Finish: BZ-3.
- Problem 4ab, exam March 21, 2003 (integration, convolution).
- Problem 4, exam March 26, 2004 (distribution theory).
- Problem 4bd, exam March 17, 2006 (convolution, inner product).
- Problem 3b, exam January 17, 2007 (convolution).
- Problem 2, exam April 29, 2008 (algebra).
- Prolem 2, exam June 13, 2008 (algebra).
- Problem 1, exam April 7, 2011 (algebra).

✗ **C-5a:**

✗ **C-5b:**

✔ **BZ-5:**

- Finish: BZ-4.
- Problem 3, exam July 8, 2004 (distribution theory).
- Problem 4, exam March 8, 2005 (distribution theory).
- Problem 4, exam June 14, 2005 (distribution theory).
- Problem 4, exam January 17, 2007 (distribution theory).
- Problem 4, exam March 21, 2007 (distribution theory).
- Problem 3, exam April 22, 2009 (distribution theory).
- Problem 3, exam June 15, 2009 (distribution theory).

- Problem 3, exam January 17, 2011 (distribution theory).
- Problem 2, exam April 7, 2011 (distribution theory).

✗ **C-6a:**

✗ **C-6b:**

✔ **BZ-6:**

- Finish: BZ-5.
- Problem 4cdef, exam March 21, 2003 (Fourier theory)
- Problem 3, exam July 11, 2003 (Fourier theory)
- Problem 3, exam March 26, 2004 (Fourier theory)
- Problem 4, exam July 8, 2004 (Fourier theory)
- Problem 3, exam June 14, 2005 (Fourier theory)
- Problem 4ac, exam March 17, 2006 (Fourier theory)
- Problem 5, exam June 28, 2006 (Fourier theory)
- Problem 3a(b)cd, exam January 17, 2007 (Fourier theory)
- Problem 3, exam March 21, 2007 (Fourier theory)
- Problem 4, exam April 29, 2008 (Fourier theory)
- Problem 4, exam June 13, 2008 (Fourier theory)
- Problem 4, exam June 15, 2009 (Fourier theory)
- Problem 4, exam January 17, 2011 (distribution theory)

✗ **Recommendation for X-Mas Holidays:**

- BZ-1/6, v.s.
- Integral exam January 23, 2012 (max. 3 hours)
- Integral exam January 25, 2013 (max. 3 hours)
- Integral exam January 24, 2014 (max. 3 hours)
- Integral exam January 23, 2015 (max. 3 hours)

✗ **C-7a:**

- Exam January 23, 2012.

✗ **C-7b & BZ-7:**

- Exam January 24, 2014.
- Q&A.

✗ **C-8a:**

- Q&A.

✗ **C-8b & BZ-8:** *Cancelled*.

Luc Florack, Eindhoven, Friday 22nd November, 2019.