MATHEMATICAL TECHNIQUES FOR IMAGE ANALYSIS
HOMEWORK ASSIGNMENT

Course code: 8D020. Teacher: Dr L.M.J. Florack, WH 3.108 (secretariat WH 2.106), **E** L.M.J.Florack@tue.nl, **T** 040 2475377, **F** 040 2472740, **W** www.bmi2.bmt.tue.nl/image-analysis/people/lflorack

Read this first!

- Make this assignment by yourself or together with *maximally* one fellow student that has also subscribed for this course.
- Write your name(s) and student number(s) on each sheet.
- The deadline for handing in this assignment is *Wednesday November 22 2006.* Assignments arriving after this date will be ignored.
- This assignment will be evaluated with a grade between 0 and 1. This is the bonus that will be added to your (re)examination grade in 2007. (The final grade cannot be higher than 10.)
- Provide clear arguments, and write neatly. Illegible or sloppy formulations will not be corrected. Explain conceptual steps in your proofs.

Problem 1. We define the *hyperbolic sine and cosine functions* as follows:

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh x = \frac{e^x - e^{-x}}{2} \quad (x \in \mathbb{R})\,.$$

($\frac{1}{10}$) **a.** Prove the following identities:

$$\cosh(\xi+\eta) - \cosh(\xi-\eta) = 2\sinh\xi\,\sinh\eta\,, \tag{1}$$
$$\cosh(\xi+\eta) + \cosh(\xi-\eta) = 2\cosh\xi\,\cosh\eta\,, \tag{2}$$
$$\sinh(\xi+\eta) - \sinh(\xi-\eta) = 2\cosh\xi\,\sinh\eta\,, \tag{3}$$
$$\sinh(\xi+\eta) + \sinh(\xi-\eta) = 2\sinh\xi\,\cosh\eta\,. \tag{4}$$

We define the set of real-valued 2×2-matrices

$$G = \left\{ \left(\begin{array}{cc} \cosh\xi & \sinh\xi \\ \sinh\xi & \cosh\xi \end{array} \right) \,\middle|\, \xi \in \mathbb{R} \right\}\,,$$

and endow it with a product operation in the usual way, i.e. standard matrix multiplication.

b. Show that G constitutes a group. To this end, answer the following questions, and provide proofs for your answers:

($\frac{1}{10}$) **b1.** Is G closed under matrix multiplication? In other words, does $A, B \in G$ imply $A\,B \in G$?

($\frac{1}{10}$) **b2.** Show *for general $n\times n$-matrices* that matrix multiplication is associative.

($\frac{1}{10}$) **b3.** Give the identity element of G.

($\frac{1}{10}$) **b4.** Give the inverse element $A^{-1} \in G$ for given $A \in G$.

$(\frac{1}{10})$ **c** Is G commutative?

Problem 2. In this problem G and H are two given groups. The infix product operator of G is indicated by a $\bullet$, whereas that of H is denoted by $\circ$. We construct the set F as follows

$$F = G \times H \stackrel{\text{def}}{=} \{(g,h) \mid g \in G, h \in H\} ,$$

which is endowed with an infix product operator $\star$ as follows. If $f_1, f_2 \in F$, say $f_1 = (g_1, h_1)$ and $f_2 = (g_2, h_2)$ with $g_1, g_2 \in G$ and $h_1, h_2 \in H$, then

$$f_1 \star f_2 = (g_1 \bullet g_2, h_1 \circ h_2) .$$

a. Show that F constitutes a group. To this end, answer the following questions, and provide proofs for your answers:

$(\frac{1}{10})$ **a1.** Is F closed under $\star$?

$(\frac{1}{10})$ **a2.** Show that the operator $\star$ satisfies the associativity property.

$(\frac{1}{10})$ **a3.** Give the identity element of F.

$(\frac{1}{10})$ **a4.** Give the inverse element $f^{-1} \in F$ for given $f \in F$.

THE END