Regression Analysis and Analysis of Variance

Exercises 2

Exercise 2.1
Let $F$ be an arbitrary distribution function. For $p$, $0 < p < 1$, define

$$F^{-1}(p)) = \inf \{ x : p \leq F(x) \}$$

Show

(a) If $U$ is uniformly distributed over $[0, 1]$ then $X = F^{-1}(U)$ has the distribution function $F$.

(b) If $F$ is continuous in $F^{-1}(p)$ then $F(F^{-1}(p)) = p$.

Exercise 2.2
Let $(X_i)_{i=1}^n$ be i.i.d. random variables with continuous distribution function $F$ and empirical distribution function

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} \{ X_i \leq x \}.$$ 

Show that the distribution of

$$d_{ko}(F_n, F) = \max_{x} | F_n(x) - F(x) |$$

is independent of $F$:

Exercise 2.3
Write an R-macro to calculate the value of the M-estimator $T_L(P_n)$

$$\sum_{i=1}^{n} \psi(x_i - T_L(P_n)) = 0.$$ 

Use the initial guess $m_0 = \text{med}(x_n)$. 
Exercise 2.4
Write an R-macro to calculate the value of the M-estimator $T_S(\mathbb{P}_n)$

$$
\sum_{i=1}^{n} \chi(x_i/T_S(\mathbb{P}_n)) = 0.
$$

Use the initial guess $s_0 = \text{MAD}(x_n)$.

Exercise 2.5
Write an R-macro to calculate the value of the M-estimator $(T_L(\mathbb{P}_n), T_S(\mathbb{P}_n))$

$$
\sum_{i=1}^{n} \psi \left( \frac{(x_i - T_L(\mathbb{P}_n))}{T_S(\mathbb{P}_n)} \right) = 0 \quad (1)
$$

$$
\sum_{i=1}^{n} \chi \left( \frac{(x_i - T_L(\mathbb{P}_n))}{T_S(\mathbb{P}_n)} \right) = 0
$$

Start the iteration with (1) with $s_0 = \text{MAD}(x_n)$. 