Regression Analysis and Analysis of Variance

Exercises 3

Exercise 3.1
For $N(0, 1)$–samples $X_n = (X_1, \ldots, X_n)$ of sizes

$$n = 3(1)20, 25(5)50, 60(10)100, 200(100)500, 1000$$

use simulations to calculate the 0.95– and 0.99–quantiles of the two statistics

$$OR_1 = \max_i \{|X_i - \text{med}(X_n)|/\text{MAD}(X_n)\}$$
$$OR_2 = \max_i \{|X_i - \text{shorth}(X_n)|/\text{lshorth}(X_n)\}.$$

What can you say about the asymptotic behaviour of the quantiles?

Exercise 3.2
Calculate the finite sample breakdown point of the interquartile range as a scale functional.

Exercise 3.3
Let $\psi(x) = (\exp(cx) - 1)/(\exp(cx) + 1)$ and $\chi(x) = (x^4 - 1)/(x^4 + 1)$. For which values of $c$ is the condition $\psi \chi 1$ of Theorem 1.27 of the lecture notes fulfilled.

Exercise 3.4
Let $\psi(x) = (\exp(cx) - 1)/(\exp(cx) + 1)$ and $\chi(x) = (x^4 - 1)/(x^4 + 1)$. Calculate a lower bound for the breakdown point of the location functional as a function of $c$.

Exercise 3.5
Generate a $N(0, 1)$ sample of size $n$ $(X_1, \ldots, X_n$ with say $n = 10$, an calculate the mean, median, shorth and the joint location-scale M-functional with $\psi(x) = (\exp(5x) - 1)/(\exp(5x) + 1)$ and $\chi(x) = (x^4 - 1)/(x^4 + 1)$. Now increase the value of the first observation $X_1$ by 10 in steps of 0.1 and plot the values of the four location functionals. Repeat for the first two, three four and five observations. Repeat for large sample sizes. Can you draw any conclusions?