A MATRIX GEOMETRIC APPROACH FOR RANDOM WALKS

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The objective of this work is to demonstrate how to obtain the equilibrium distribution of the state of a two-dimensional homogeneous nearest neighbour (simple) random walk restricted on the lattice using the matrix geometric approach. This type of random walks can be modelled as a QBD process with the characteristic that both the levels and the phases are countably infinite. Then, based on the matrix geometric approach, if \( \pi_n = (\pi_{n,0}, \pi_{n,1}, \ldots) \) denotes the vector of the equilibrium distribution of level \( n \), \( n = 0, 1, \ldots \), it is known that \( \pi_{n+1} = \pi_n R \). Although, this is a very well known result, the complexity of the solution lies in the calculation of the infinite dimension matrix \( R \). We will demonstrate a new methodological approach for the direct calculation of the eigenvalues and eigenvectors of matrix \( R \).

This work promises 1) a wide spectrum of applicability 2) an easy theoretical framework, while also promising 3) the unification of three existing approaches for random walks (matrix geometric approach; compensation approach; boundary value problem), as well as 4) the first steps towards the probabilistic interpretation of the underlying terms involved in the solution.