EXACT FINITE APPROXIMATIONS OF AVERAGE-COST COUNTABLE MARKOV DECISION PROCESSES

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For a countable-state, compact action space Markov decision process we introduce a method to derive embeddings which produce finite-state Markov decision processes. Each finite-state embedded process has the same optimal cost as the original model—and similarly, costs of any stationary Markov policy agree. Moreover, the embedded Markov decision process agrees with the original process when restricting to the approximating finite set, in the sense that the available actions are the same, and the use of corresponding actions yields identical transition probabilities and identical immediate rewards.

The embedded process can be used as an approximation which, being finite, is more convenient for computation and implementation. Since the optimal costs are the same, it is an approximation only in the sense that the process is different outside the finite set. The size of the approximation determines the range of states on which the approximation has the same dynamics as the original process.

This method extends easily to constrained MDPs. The ideas also apply to optimal exit problems.