A stationary sequence of random variables with regularly varying tails is considered. For this sequence it is possible that large values arrive in clusters. That is, there may be many large values in a relatively short period of time. The aim is to give a detailed description of the occurrence of large values, not only the size of clusters but also within a single cluster. To do this a point process based on appropriately scaled points of the stationary sequence is constructed. A limiting measure, on the space of point measures, describes the joint limiting behavior of all the large values of the sequence. From this limiting result one can proceed and obtain the functional large deviation for its partial sums, ruin probabilities, etc. Examples include, linear processes, random coefficient ARMA processes, and solutions to stochastic recurrence equations.