Extremes of univariate heavy-tailed first-order Markov chains are known to behave under general conditions as a multiplicative random walk, the tail chain. This property is found to be a particular case of a much more general one stating that a stationary multivariate time series is multivariate regularly varying if and only if the following property holds: conditionally on the value of the process at a fixed time point being large (in norm), the process converges in the sense of finite-dimensional distributions to a limit process, the tail process. This tail process is found to possess a number of remarkable properties. Moreover, all kinds of interesting quantities can be expressed in terms of this tail process: limit point processes, multivariate extremal indices, functionals of clusters of multivariate extremes. For general multivariate Markov chains, the tail process takes a special form, yielding neat formulas in certain examples, for instance for solutions to stochastic difference equations or moving averages with random coefficients.