A ONE-DIMENSIONAL DIFFUSION MODEL FOR OVERLOADED QUEUES WITH CUSTOMER ABANDONMENT

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We use an Ornstein–Uhlenbeck (OU) process to approximate the queue length process for a GI/GI/n + M queue. This simple one-dimensional diffusion model is able to produce accurate performance estimates in two overloaded regimes: In the first regime, the number of servers is large and the mean patience time is comparable or longer than the mean service time; in the second regime, the number of servers can be arbitrary but the mean patience time is much longer (i.e., on a higher order) than the mean service time. We formulate these two regimes into an asymptotic framework where a sequence of queues is considered. The mean patience time goes to infinity in both asymptotic regimes, whereas the number of servers approaches infinity in the first regime but does not change in the second. The OU process is proved to be the diffusion limit for queue length processes in both asymptotic regimes. A crucial tool for proving the diffusion limit is a functional central limit theorem for the superposition of renewal processes. We prove that the superposition of n independent, identically distributed stationary renewal processes, after being centered and scaled in both space and time, converges to a Brownian motion as n goes large.