RARE-EVENT PROBABILITY ESTIMATION VIA EMPIRICAL LIKELIHOOD

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Consider the problem of estimating the probability that the sum of d correlated lognormal random variables exceeds a given large threshold γ. In other words, consider estimating

$$\ell = \mathbf{P}(\mathsf{e}^{X_1} + \cdots + \mathsf{e}^{X_d} > \gamma) \quad , \quad (X_1, \ldots, X_d) \sim \mathsf{N}(\mathbf{0}, \Sigma),$$

where $\mathsf{N}(\boldsymbol{\mu}, \Sigma)$ denotes the multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix Σ. This problem arises in some risk insurance models. We describe a new Monte Carlo method for estimating ℓ, in which ℓ is part of a nonparametric (empirical) maximum likelihood estimation problem. We provide numerical results that show that the currently used estimators of ℓ give inaccurate results, even though they have been shown to have vanishing relative error properties. We thus argue that vanishing relative error estimators can sometimes perform poorly in practice, and that our proposed algorithm is the first to provide an accurate and reliable estimator of ℓ.