JOIN THE SHORTEST QUEUE NETWORKS: THEIR LIMITING BEHAVIOR AND OPEN PROBLEMS

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In join the shortest queue networks, incoming jobs are assigned to the shortest queue from among a randomly chosen subset of $D$ queues in a system of $N$ queues. After completion of service at its queue, a job leaves the system. We assume that jobs arrive into the system according to a rate $-\alpha N$ Poisson process, $\alpha < 1$, and that service at each queue is at rate 1.

In the fundamental paper of Vvedenskaya, Dobrushin, and Karpelevich (1996), it was shown that, if the service times at each queue are exponentially distributed, then the right tail of the equilibrium distribution at each queue decays doubly exponentially in the limit, as $N$ goes to infinity. This is a substantial improvement over the case $D = 1$, where the queue size decays exponentially; this faster decay is important for various applications.

When the assumption of exponential service times is dropped, analysis of the problem becomes considerably more complicated, with significant aspects remaining to a large extent open. In recent work, the speaker, together with Y. Lu and B. Prabhakar, has investigated the asymptotic behavior of the tails of equilibrium distributions when service is FIFO and the service time distributions are Pareto. There, it was shown that, depending on the exact parameters that are involved, the tails can decay either polynomially, exponentially, or doubly exponentially fast. A number of other situations were also investigated.

A crucial step in the analysis of the equilibrium distributions is to show asymptotic independence of the individual queues, as $N$ goes to infinity. While "obviously" true in a large variety of cases, this has been shown in only more limited contexts. In this talk, background for join the shortest queue networks will be given, including a summary of the above results. Asymptotic independence and certain other open problems will be discussed.