FROM LOCAL TO GLOBAL STABILITY IN STOCHASTIC PROCESSING NETWORKS THROUGH QUADRATIC LYAPUNOV FUNCTIONS

A.B. Dieker, Georgia Institute of Technology, USA, ton.dieker@isye.gatech.edu
J. Shin, IBM Research, USA, jshin@us.ibm.com

We construct a generic, simple, and efficient scheduling policy for stochastic processing networks, and provide a general framework to establish its stability. Our policy is randomized and prioritized: with high probability it prioritizes jobs which have been least routed through the network. We show that the network is globally stable under this policy if there exists an appropriate quadratic 'local' Lyapunov function that provides a negative drift with respect to nominal loads at servers. Applying this generic framework, we obtain stability results for our policy in many important examples of stochastic processing networks: open multiclass queueing networks, parallel server networks, networks of input-queued switches, and a variety of wireless network models with interference constraints. Our main novelty is the construction of an appropriate 'global' Lyapunov function from quadratic 'local' Lyapunov functions, which we believe to be of broader interest.