We introduce a new partial order on the class of stochastically monotone Markov kernels having a given stationary distribution $\pi$ on a given finite partially ordered state space $X$. When $K \preceq L$ in this partial order we say that $K$ and $L$ satisfy a comparison inequality. We establish that if $K_1, \ldots, K_t$ and $L_1, \ldots, L_t$ are reversible and $K_s \preceq L_s$ for $s = 1, \ldots, t$, then $K_1 \cdots K_t \preceq L_1 \cdots L_t$. In particular, in the time-homogeneous case we have $K^t \preceq L^t$ for every $t$ if $K$ and $L$ are reversible and $K \preceq L$, and using this we show that (for suitable common initial distributions) the Markov chain $Y$ with kernel $K$ mixes faster than the chain $Z$ with kernel $L$, in the strong sense that at every time $t$ the discrepancy—measured by total variation distance or separation or $L^2$-distance—between the law of $Y_t$ and $\pi$ is smaller than that between the law of $Z_t$ and $\pi$.

Using comparison inequalities together with specialized arguments to remove the stochastic monotonicity restriction, we answer a question of Persi Diaconis by showing that, among all symmetric birth-and-death kernels on the path $X = \{0, \ldots, n\}$, the one (we call it the uniform chain) that produces fastest convergence from initial state 0 to the uniform distribution has transition probability 1/2 in each direction along each edge of the path, with holding probability 1/2 at each endpoint.

We also use comparison inequalities

(i) to identify, when $\pi$ is a given log-concave distribution on the path, the fastest-mixing stochastically monotone birth-and-death chain started at 0, and

(ii) to recover and extend a result of Peres and Winkler that extra updates do not delay mixing for monotone spin systems.

Among the fastest-mixing chains in (i), we show that the chain for uniform $\pi$ is slowest in the sense of maximizing separation at every time.