Tempered stable distributions were introduced in Rosiński 2007 as models that look like infinite variance stable distributions in some central region, but they have lighter (i.e. tempered) tails. Such models have found applications in a variety of areas including mathematical finance, biostatistics, computer science, and physics. We extend this class to allow for more variety in the tails. While some cases no longer correspond to stable distributions they serve to make the class more flexible, and in certain subclasses they have been shown to provide a good fit to data. To characterize the possible tails we give detailed results about finiteness of various moments. We also give necessary and sufficient conditions for the tails to be regularly varying. This last part allows us to characterize the domain of attraction to which a particular tempered stable distribution belongs. We then characterize the weak limits of sequences of tempered stable distributions. We will conclude by discussing a mechanism by which distributions that are stable-like in some central region but with lighter tails show up in applications.