DIFFUSION MODELS AND STEADY-STATE APPROXIMATIONS FOR EXPONENTIALLY ERGODIC MARKOVIAN QUEUES

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Motivated by queues with many-servers, we study Brownian steady-state approximations for continuous time Markov chains (CTMCs). Our approximations are based on diffusion models (rather than a diffusion limit) whose steady-state, we prove, approximates well that of the Markov chain. Strong approximations provide such "limitless" approximations for process dynamics. Our focus here is on steady-state distributions and the diffusion model that we propose is tractable relative to strong approximations. Within an asymptotic framework, in which a scale parameter n is taken large, a uniform (in the scale parameter) Lyapunov condition is proved to guarantee that the gap between steady-state moments of the diffusion and those of the properly centered and scaled CTMCs, shrinks at a rate of $\sqrt{n}$. The uniform Lyapunov requirement is satisfied, in particular, if the scaled and centered sequence converges to a diffusion limit for which a Lyapunov condition is satisfied. Our proofs build on gradient estimates for the solutions of the Poisson equations associated with the (sequence of) diffusion models together with elementary Martingale arguments. As a by product of our analysis, we explore connections between Lyapunov functions for the Fluid Model, the Diffusion Model and the CTMC.