LIMIT THEOREMS FOR SMOLUCHOWSKI DYNAMICS ASSOCIATED WITH CRITICAL CONTINUOUS-STATE BRANCHING PROCESSES

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We will exploit a well known connection (Bertoin, Le Gall '06) between critical continuous-state branching processes (CSBP) that are absorbed at 0 and the generalized Smoluchowski coagulation equation to prove a variety of limit theorems. We establish a fundamental connection between the existence of a non-degenerate scaling limit and regular variation (at 0) of the branching mechanism. In particular, we show:

- A non-degenerate scaling limit of a critical CSBP (absorbed at 0) exists if and only if the branching mechanism is regularly varying at 0.

- The CDF of the rescaled process conditioned on survival has a nondegenerate limit if and only if the branching mechanism is regularly varying at 0.

When the scaling limits exist, we can further characterise the nondegenerate scaling limits of arbitrary finite-measure solutions in terms of generalized Mittag-Leffler series.