Differential equation is an important tool to describe deterministic systems. In practice, systematic and/or random errors often exist such that some coefficients are not measured precisely, which induces uncertainties and sometimes puts the entire system in risk or under a failure situation. In this talk, we consider the extreme behavior and rare-event simulation problems for some of these random differential equations whose coefficients are described by spatially varying processes. This analysis has its applications in material science, fluid dynamics, ocean-earth sciences, etc.