Exploiting Harmonic Periods to Improve Linearly Approximated Response-Time Upper Bounds
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Abstract
Exact schedulability tests for fixed-priority preemptively scheduled systems are pseudo-polynomial in complexity. A linear-time sufficient test [1,2] has therefore been developed to estimate response-time upper bounds. In line with utilization-based sufficient tests, we propose to improve this test for task sets with harmonically related task periods. Moreover, we make it possible to reuse this test in the context of hierarchically scheduled resources. In such systems several applications are given a virtual share (budget) of the processor. By modeling the unavailability of processor resources to an application as two fictive tasks, we can also use a budget’s period to improve response-time bounds.

1. Existing Response-time Upper Bounds
We consider task set mapped on a single, shared, fixed priority preemptively scheduled resource. The approach presented in [1, 2] assumes that the entire processor is available to a task set.

The response-time upper bounds, $R$, derived in [1,2] work as follows:
1. linear approximation of the interference, $R^{UB}_{t_k}$, of each higher priority task, $\tau_i$, is derived;
2. these linear approximations are summed up and the computation time of the task is added; and
3. the intersection of the resulting equation with the processor supply is calculated.

The interference of a single task over a time interval of length $t$ is defined:
$$I^{UB}_{t_k}(t) = U^*_t + C(t - U^*_t).$$

The linear processor supply over a time interval $t$ is given by
$$y = t.$$ 

An upper bound on the worst-case response time, $R^{UB}$, of task $\tau_i$ is [1]:
$$R^{UB}_{t_k} = \left( \sum_{j=1}^{k} C_j (1 - U^*_t) / (1 - \sum_{j=1}^{k} U^*_j) \right).$$

In our paper we do not consider activation jitter. The above derivation is shown in Figure 1 for an example task set, see Table I.

<table>
<thead>
<tr>
<th>Task</th>
<th>Period (deadline)</th>
<th>Computation time</th>
<th>$R^*_{t_k}$ (exact)</th>
<th>$R^{UB}_{t_k}$ [1]</th>
<th>$R^{UB}_{t_k}$ (new)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>200</td>
<td>75</td>
<td>150</td>
<td>155</td>
<td>155</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>975</td>
<td>100</td>
<td>800</td>
<td>1195</td>
<td>975</td>
</tr>
</tbody>
</table>

2. EDP resource unavailability
We showed in [4] that we can reuse the method in [1, 2] on a shared EDP resource [3], $\Omega$, by substituting the linear processor supply $y=t$ with a (tight) lower bound the EDP supply bound function $lb_{p_{ij}}(t)$.

We can alternatively model the unavailability of a partitioned EDP resource by two fictive tasks at the highest priority. These two tasks have the same period and execute in distinct periods, i.e. they do not interfere with each other.

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3. Tangent of the combined task
We can construct an artificial task which represents the combined workload of a set of higher-priority, harmonic tasks. Its period equals the hyper-period of all harmonic tasks, and its computation time is the sum of all job executions that can occur in such an interval. Figure 2 illustrates this approach on the example task set of Table I.

![Figure 1: Davis and Burns’ [1] approach.](image1)

![Figure 2: Tangent-of-the-combined-task approach.](image2)

Given two harmonic tasks with $T_2 = k^* T_1$ and $k \in \mathbb{N}^+$, our approach:
- dominates the approach in [1] if $C_2 > \frac{1}{k} (k-1) (T_1 - C_1)$;
- gives the tightest possible linear upper bound if $(k-1) C_1 + C_2 \geq (k-1) T_1$.

Both properties apply to the two fictive tasks that model the unavailability of an EDP resource. As a result, when the period of a task is harmonic with its budget, our unavailability approach together with the tangent-of-combined-tasks approach may lead to improved response-time upper bounds compared to the straightforward method using $lb_{p_{ij}}(t)$.

Future Work
- Extend our analysis for tasks with activation jitter;
- Compare our approach with utilization-based schedulability tests;
- Exploit harmonic periods to efficiently calculate EDP budget parameters.

References