1.1 CTL model checking

Consider the following transition system over $AP = \{\text{black, green, red, yellow}\}$.

![Transition System Diagram]

Indicate the satisfaction set for each of the following CTL formulas. The computation should be shown. Also give an intuitive meaning of each formula.

(a) $\forall (\Diamond \text{yellow})$
(b) $\forall (\Box \text{yellow})$
(c) $\forall (\Box (\forall (\Diamond \text{yellow})))$
(d) $\forall (\Diamond \text{green})$
(e) $\exists (\Diamond \text{green})$
(f) $\exists (\Box \text{green})$

(g) $\exists (\Box \neg \text{green})$
(h) $\forall (\text{black} U \neg \text{black})$
(i) $\exists (\text{black} U \neg \text{black})$
(j) $\forall (\neg \text{black} U \text{black})$
(k) $\exists (\neg \text{black} U \text{black})$

1.2 CTL Parse Trees

The following transition system over $AP = \{a, b\}$ is given:

Consider the CTL formula $\Phi = \forall (\neg (\exists (\Box b) \lor \neg \exists (Xa)) \lor \forall (X(\neg \forall (\Diamond b))))$

a) Give the parse tree of $\Phi$.

b) Determine the satisfaction set of $\Phi$.

1.3 Umbrella Problem

An absent-minded professor has 2 umbrellas that she uses when commuting from home to office and back. If it rains and an umbrella is available in her location, she takes it. If it is not raining, she always forgets to take an umbrella. Suppose that it rains with probability 0.6 each time she commutes, independently of other times. Our goal is to find the fraction of days she gets wet during a commute.

To answer this question we want to use a DTMC model where a state represents the number of umbrellas at her current location. A transition in the DTMC corresponds to one commute of the professor.

a) Draw the state-transition diagram of the DTMC modelling the Umbrella Problem.

b) Derive the corresponding transition probability matrix.

c) Compute the steady-state distribution.

d) Compute the long-term fraction of commutes she gets wet.

e) If before commuting in the morning she has one umbrella at her disposal, what is the probability to have two umbrellas in the evening (after commuting)?
1.4 Classification of states

Consider the following DTMC (the initial state is 0):

![Diagram of DTMC](image.png)

a) What is the state space $S$ of this DTMC? Also derive the transition probability matrix $P$.

b) A state $j$ is said to be accessible from state $i$ if, for some value $n \geq 0$, $p_{i,j}(n) > 0$. For such a pair of states we write $i \rightarrow j$. If $i \rightarrow j$ and $j \rightarrow i$, then $i$ and $j$ are said to be communicating states, denoted $i \sim j$. The relation $\sim$ is an equivalence relation. Determine the equivalence classes of the state space. Which of them contain transient states, which recurrent states?

c) This DTMC is not irreducible. However, there is one irreducible equivalence class, the DTMC will eventually end up in. Which class is this?

d) It is possible to calculate the steady-state probabilities just for this class. Why? Calculate them by hand.
1.5 PRISM and DTMCs

(Do this exercise at home, as preparation for the assignment!)
Consider the following DTMC (the state $s_5$ is the initial state):

![Diagram of DTMC]

a) Download and install PRISM (see www.prismmodelchecker.org).
b) Start PRISM. Model the system described above using the PRISM language (see http://www.prismmodelchecker.org/tutorial/die.php for a very helpful example).
c) By hand, compute the steady-state probability of being in state $s_3$.
d) We will now obtain the same result using PRISM. Go to the ‘Model’ drop down menu, and select ‘Build Model’. Then use ‘Compute’ to compute the steady-state probabilities. Check whether the result matches with your own calculations.
e) Remove the transition from $s_3$ to $s_5$ and set the probability of jumping from $s_3$ to $s_2$ to equal 1. Tell PRISM to recompute the steady-state probabilities. What is the result? Explain.
f) Now add self-loops with probability 1 to states $s_1$ and $s_3$. In what way do you have to change the probabilities of the other outgoing transitions from these states? Calculate the limiting probability of being in state $s_3$ by hand, starting in state $s_5$.
g) Tell PRISM to do the same. Compare your results. Do the limiting probabilities depend on the initial state? Why/why not?
h) In the original DTMC, compute by hand the probability of jumping from state $s_2$ to state $s_4$ in exactly five steps.
i) Do the same in PRISM. Compare your results.