

# EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Mathematics and Computer Science

*Examination Real-time Architectures (2IN25)  
on Tuesday, January 6<sup>th</sup> 2009, 14.00h-17.00h.*

First read the entire examination. There are 5 exercises in total. Grades are included between parentheses at all parts and sum up to 9 points. Good luck!

1. The hyperbolic bound  $HB(n) : \prod_{1 \leq i \leq n} (U_i^\tau + 1) \leq 2$ , where  $U_i^\tau$  denotes the processor utilization of the task  $\tau_i$ , is an example of a *sufficient* schedulability test.
  - (a) (0.5) What can be concluded concerning schedulability of a set  $\mathcal{T}$  of  $n$  task when the hyperbolic bound does not hold for  $\mathcal{T}$ ?  
**Answer:** The task set  $\mathcal{T}$  may but need not be schedulable. Hence, nothing can be concluded yet...
  - (b) (1.0) Construct an example set of three tasks for which the left-hand side of  $HB(3)$  is equal to 2.  
**Answer:** See RTA.Rehearse-080918.
  - (c) (0.5) Is the task set that you constructed schedulable according to the Liu & Layland bound? Motivate your answer.  
 Reminder:  $LL(n) : U^T \leq n(2^{1/n} - 1)$ , where  $U^T$  denotes the processor utilization of the set  $\mathcal{T}$ .  
**Answer:** The Liu & Layland bound will also hold if and only if the utilizations of all three tasks are the same; see RTA.Exercises-2 "Solutions LL-bound". Because the Liu & Layland bound is also a sufficient condition, just like the hyperbolic bound, nothing can be concluded when the bound does not hold.
2. Consider two periodic tasks  $\tau_1$  and  $\tau_2$ , with characteristics as given in the following table, which are scheduled by means of fixed-priority scheduling with deferred pre-emption (FPDS), where  $\tau_1$  has a higher priority than  $\tau_2$ .

	$T = D$	$C$
$\tau_1$	3	$1\frac{1}{2}$
$\tau_2$	5	$1 + 1\frac{1}{4}$

- (a) (1.0) Determine the worst-case length  $WL_2$  of the level-2 active period.  
**Answer:** Draw a timeline with a simultaneous release of both tasks at time  $t = 0$ . Because the pending load becomes zero at time  $t = 9$ , the worst-case length  $WL_2 = 9$ .
- (b) (1.0) Determine the worst-case response times  $WR_1^D$  of  $\tau_1$  and  $WR_2^D$  of  $\tau_2$  using

$$WR_{ik}^D = \begin{cases} WR_i^P(B_i^D + kC_i - F_i) + F_i - (k-1)T_i & \text{for } i < n \\ WO_i^P(kC_n - F_n) + F_n - (k-1)T_n & \text{for } i = n \end{cases}, \quad (1)$$

and the following recursive equations for  $WR_i^P(C)$  and  $WO_i^P(C)$ , respectively.

$$x = C + \sum_{j < i} \left\lceil \frac{x}{T_j} \right\rceil C_j, \quad (2)$$

$$x = C + \sum_{j < i} \left( \left\lceil \frac{x}{T_j} \right\rceil + 1 \right) C_j. \quad (3)$$

**Answer:** For task  $\tau_1$ , it is only necessary to consider the first job:  $WR_1^D = 2\frac{3}{4}$ . Because the worst-case length  $WL_2$  of the level-2 active period contains  $\lceil \frac{WL_2}{T_2} \rceil = 2$  jobs, we need to consider two jobs for task  $\tau_2$ .  $WR_{2,1}^D = 3\frac{3}{4}$  and  $WR_{2,2}^D = 4$ , hence  $WR_2^D = 4$ .

3. Consider four periodic tasks  $\tau_1, \tau_2, \tau_3$  and  $\tau_4$  (having decreasing priority), which share five resources,  $A, B, C, D$ , and  $E$ . Compute the maximum blocking time  $B_i$  for each task for the following three protocols, knowing that the longest duration  $D_i(R)$  for a task  $\tau_i$  on resource  $R$  is given in the following table (there are no nested critical sections).

	$A$	$B$	$C$	$D$	$E$
$\tau_1$	6	7	0	10	3
$\tau_2$	0	0	0	8	0
$\tau_3$	4	14	8	0	0
$\tau_4$	0	11	0	9	7

- (a) (1.0) Priority Inheritance Protocol (PIP).

**Answer:** Similar to Exercise 7.5 of the book of Buttazzo. Compared to that exercise, the columns have been exchanged ( $A \rightarrow E \rightarrow B \rightarrow A$  and  $C \leftrightarrow D$ ), and all non-zero values have been increased by 1.

- (b) (0.5) Priority Ceiling Protocol (PCP).

**Answer:** Similar to Exercise 7.6 of the book of Buttazzo.

- (c) (0.5) Highest Locker Protocol (HLP).

**Answer:** Same as for PCP.

4. The following questions concern presentations of assignments.

- (a) (0.5) *Hierarchical scheduling of independent applications*: Two approaches were presented for two-level hierarchical scheduling of independent applications, one termed "Hierarchical Fixed-Priority Pre-emptive Scheduling" by Davis and Burns and the other "Periodic resource model for compositional real-time guarantees" by Shin and Lee. Although the analysis for the 2<sup>nd</sup> approach is more pessimistic than the former, the analysis for 2<sup>nd</sup> approach also has a major advantage. Explain that advantage.

**Answer:** Given the parameters of its server, the schedulability of a subsystem can be analyzed in isolation, i.e. without any knowledge about periods or capacities of other servers.

- (b) *Hierarchical scheduling of dependent applications*: Two approaches were presented for global resource access, one approach termed H-SRP and the other termed SIRAP. The approaches differed with respect to their behavior when the amount of remaining capacity of a reservation was less than the amount of time needed for accessing the global resource.

- i. (0.5) Explain the difference between both approaches.

**Answer:** H-SRP assumes budget *overrun* (optionally with *pay-back*) and SIRAP assumes (*self-*) *blocking*.

- ii. (0.5) What are the consequence of SIRAP for a supporting mechanism?

**Answer:** Upon entering a critical section, a check is needed whether or not there is still enough remaining capacity to use the global resource, i.e. to execute the critical section.

5. The following questions concern guest-lectures.

- (a) (0.5) Prof. Christian Hentschel gave a guest lecture entitled *Scalable video algorithms*. Briefly describe his motivation for scalable video algorithms.

**Answer:** See his slides.

- (b) (0.5) Mike Holenderski gave a guest lecture entitled *Swift mode changes*. He considered two scheduling approaches for mode changes, fixed-priority pre-emptive scheduling (FPPS) and fixed-priority scheduling with deferred pre-emption (FPDS). Which approach is better and why?

**Answer:** Assuming that processing a mode corresponds with a single sub-job, the worst-case lead-time of a mode-change is larger for FPPS than for FPDS, because for FPDS we need to wait for at most 1 sub-job to complete.

- (c) (0.5) Alina Weffers-Albu gave a guest-lecture entitled *Behavioural Analysis of Real-Time Systems with Interdependent Tasks*. She explained that the behaviour of a chain of components scheduled by means of fixed-priority pre-emptive scheduling (FPPS) assumes a repetitive pattern after an finite amount of time. How can that amount of time for the so-called initial phase be minimized?

**Answer:** By giving the first task in the chain the lowest priority and/or by minimizing the size of the buffers prior to the lowest priority component in the chain.