# EINDHOVEN UNIVERSITY OF TECHNOLOGY

**Department of Mathematics and Computer Science**

*Examination Real-time Architectures (2IN25)*
*on Friday, April* $16^{th}$ *2010, 9.00h-12.00h.*

First read the entire examination. There are 6 exercises in total. Grades are included between parentheses at all parts and sum up to 9 points. Good luck!

1. The hyperbolic bound $HB(n) : \prod_{1 \leq i \leq n}(U_i^\tau + 1) \leq 2$, where $U_i^\tau$ denotes the processor utilization of the task $\tau_i$, is an example of a *sufficient* schedulability test.

   (a) (0.5) Briefly discuss the relation between the following conditions and the *HB*-bound.

      i. Deadlines are at most equal to periods, i.e. $D_i \leq T_i$.
      **Answer**: The condition $D_i = T_i$ must hold (as a precondition) to apply the *HB*-bound. The given condition is therefore too weak for the *HB*-bound.

      ii. The sum of the utilizations is at most equal to 1.
      **Answer**: This need not hold to *apply* the *HB*-bound. When the condition does not hold, the *HB*-bound will not hold either.

   (b) (1.0) Construct an example set of four tasks for which the left-hand side of $HB(4)$ is equal to 2 and where any *increase* of any computation time or *decrease* of any period will make the task set unschedulable under RMS. *Hint*: Construct the set of tasks by means of a timeline.
   **Answer**: Construction by means of a timeline is illustrated in RTA.Exercises-2. The result satisfies the requirements that $HB(4) = 2$ and that an *increase* of any computation time makes the task set unschedulable. The result does *not* satisfy the requirement that a *decrease* of any period will make the task set unschedulable, however. Even stronger, it is possible to decrease $T_2$, $T_3$, and $T_4$ to a value equal to $T_1$, without making the task set unschedulable.

   The conclusion is therefore that this can only be done when 3 tasks have a computation time equal to zero. For $C_i > 0$, this is therefore not possible.

2. There are different types of tasks, e.g. periodic tasks with jitter, elastic tasks, and sporadic tasks.

   (a) (0.5) Describe the (distinguishing) characteristics of periodic tasks with jitter, elastic tasks, and sporadic tasks. *Hint*: Consider jitter and (best-case and worst-case) values for periods.
   **Answer**: The distinguishing characteristics are:

   - A periodic task $\tau_i$ with jitter has a fixed period $T_i$ and activation jitter $AJ_i$.
   - An elastic task $\tau_i$ has a (finite) best-case period $BT_i$, a (finite) worst-case period $WT_i$, where $WT_i \leq BT_i$, and no jitter, i.e. $AJ_i = 0$.
   - A sporadic task $\tau_i$ has an infinite best-case period $BT_i$, i.e. the inter-arrival times of its jobs may be an arbitrary amount of time apart, a (finite) worst-case period $WT_i$, and no jitter, i.e. $AJ_i = 0$.

(b) (1.0) Assume fixed-priority pre-emptive scheduling, and suppose you model a periodic task with jitter as an elastic task. Moreover, assume that the worst-case deadline is at most equal to the worst-case period minus the jitter. What would be the consequence for the calculated worst-case response time of (*i*) that task, (*ii*) tasks with a higher priority, and (*iii*) tasks with a lower priority? Motivate your answer.
**Answer**: Let the periodic task $\tau_i$ with jitter be specified with $T_i$ and $AJ_i$. It is given that $WD_i \leq T_i - AJ_i$.
The maximum and minimum inter-arrival time of the jobs of $\tau_i$ are $T_{\max} = T_i + AJ_i$ and $T_{\min} = T_i - AJ_i$, respectively. Hence, the elastic task $\tau'_i$ has a best-case period $BT'_i = T_i + AJ_i$, a worst-case period $WT'_i = T_i - AJ_i$, and no jitter, i.e. $AJ'_i = 0$. The (worst-case) deadline is assumed to remain the same, i.e. $WD'_i = WD_i \leq T_i - AJ_i = WT'_i - AJ'_i$.
We now consider the consequence for the calculated worst-case response times. Let the set of indices of tasks with a higher and lower priority than $\tau_i$ be denoted by $hp(i)$ and $lp(i)$, respectively. For ease of presentation, let's assume that the tasks are independent. Because $\tau_i$ does not influence $\tau_j$ with $j \in hp(i)$, the way $\tau_i$ is modeled does not change the calculated $WR_j$; see also (1). Similarly, none of the differences between $\tau_i$ and $\tau'_i$ changes the outcome for the task itself. Finally, because the elastic task $\tau'_i$ can give rise to more interference than the periodic task with jitter $\tau_i$ to tasks $\tau_j$ with $j \in lp(i)$, the calculated worst-case response times of those lower priority tasks can become larger.

3. Consider two periodic tasks $\tau_1$ and $\tau_2$ and a deferrable server $S_{\text{DS}}$ with characteristics as given in the following table.

| | $T = D$ | $C$ |
|---|---|---|
| $S_{\text{DS}}$ | 5 | 1 |
| $\tau_1$ | 7 | 2 |
| $\tau_2$ | 9 | 3 |

Assume scheduling based on FPPS and a rate monotonic priority assignment.

(a) (1.0) Assuming arrivals of aperiodic requests at time $t = 4$ for an amount of 1.6 and at time $t = 7$ for an amount of 2, draw time-lines illustrating the execution of the tasks and the remaining capacity of the deferrable server in an interval of length 25.
**Answer**: See Figure 1.

(b) (0.5) Are the tasks and the deferrable server schedulable under arbitrary phasing? Motivate your answer by means of *calculations*. *Hint*: Determine the worst-case response times of the tasks by means of the following recursive equation.

$$x = WC_i + \sum_{j<i} \left\lceil \frac{x + AJ_j}{WT_j} \right\rceil WC_j \tag{1}$$

**Answer**: Note that $AJ_{\text{DS}} = T_{\text{DS}} - C_{\text{DS}} = 4$. Based on (1), we find $WR_1 = 4 < D_1 = 7$ and $WR_2 = 10 > D_2 = 9$. Hence, task $\tau_2$ misses its deadline and task $\tau_2$ is therefore not schedulable.

4. Consider three tasks with arbitrary phasing and characteristics as given in the following table.

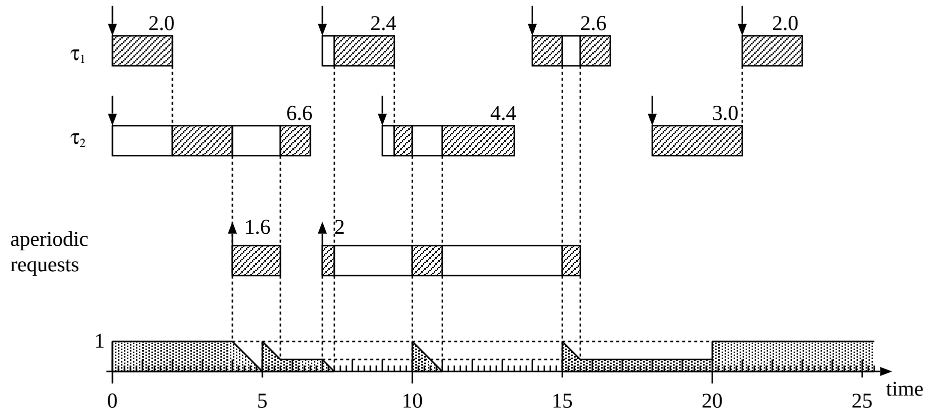


Figure 1: Example of high-priority Deferrable Server.

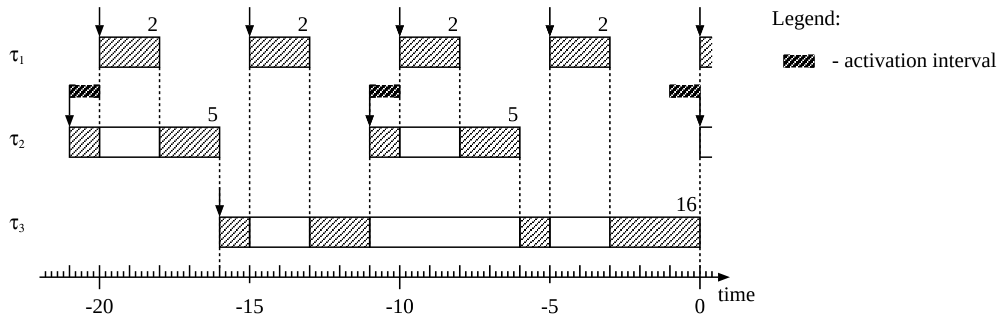


Figure 2: Time line with an optimal instant for task $\tau_3$.

| | $T$ | $WD$ | $BD$ | $AJ$ | $WC$ | $BC$ |
|---|---|---|---|---|---|---|
| $\tau_1$ | 5 | 6 | 2 | 0 | 2 | 2 |
| $\tau_2$ | 10 | 9 | 2 | 1 | 3 | 3 |
| $\tau_3$ | 37 | 35 | 15 | 0 | 9 | 7 |

(a) (0.5) Give a necessary and sufficient condition for the schedulability of the three tasks.
**Answer**: A necessary and sufficient, i.e. exact, condition is given by

$$\forall_{1 \le i \le 3} \left( BD_i \le BR_i \wedge WR_i \le WD_i \right), \tag{2}$$

where $BD_i$ and $WD_i$ are the best-case and worst-case deadline, respectively, and $BR_i$ and $WR_i$ are the best-case and worst-case response time, respectively

(b) Assume fixed-priority pre-emptive scheduling, where $\tau_1$ has highest and $\tau_3$ has lowest priority.

i. (0.5) Draw a time line with an optimal instant for task $\tau_3$.
**Answer**: See Figure 2. Note that $BR_3 = 16$.

ii. (1.0) Determine the best-case response time of task $\tau_3$ using the following recursive equation.

$$x = BC_i + \sum_{j<i} \left( \left\lceil \frac{x - AJ_j}{BT_j} \right\rceil - 1 \right)^+ BC_j \tag{3}$$

**Answer**: Using (1) we first determine the worst-case response time $WR_3 = 35 = WD_3$ as initial value for the iterative procedure to determine the best-case response time $BR_3$. Using (3), we subsequently find $BR_3 = 16 > BD = 15$.

5. The following questions concern "Expected reading" and specific lectures.

   (a) (0.5) In "I. Shin and I. Lee, *Periodic resource model for compositional real-time guarantees*, In: Proc. $24^{th}$ IEEE Real-Time Systems Symposium (RTSS), pp. 2-13, December 2003." a resource supply bound function $\texttt{sbf}_\Gamma(t)$ of a time interval of length $t$ is defined that calculates the minimum resource supply of $\Gamma$ during $t$ units. Given a periodic resource $\Gamma(\Pi, \Theta)$, draw $\texttt{sbf}_\Gamma(t)$ as a function of $t$ for $0 \leq t \leq 5\Pi$, where $\Pi/4 \leq \Theta \leq \Pi/3$.

   (b) One of the lectures concerned *A QoS approach for multimedia consumer terminals with media processing in software.* The aim of the QoS approach was *cost-effective high-quality video processing in software for multimedia consumer terminals*, motivated by the requirements for *openness* and *flexibility* of these systems, and having as boundary condition that *the existing system qualities should be preserved.*

      i. (0.5) Explain which real-time problems were addressed.

      ii. (0.5) Explain how these problems have been solved.

   **Answer** See slides of the lecture.

6. (1.0) Give an example illustrating transitive adjustment of priorities for the Priority Inheritance Protocol (PIP).
   **Answer**: See slides RTA.B4-Policies-3 and book Fig. 7.8.