Best-case response times and jitter analysis of real-time tasks with arbitrary deadlines

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Background and motivation

• Facts about fixed-priority scheduling (FPS):
  • described in standards, e.g. OSEK (Automotive);
  • supported by most COTS RTOS;
  • de facto standard in industry.

• Best-case analysis:
  • Complementary schedulability: not too early;
  • Improved finalization jitter;
  • Improved end-to-end response times [27].

Background and motivation

- **Schedulability: inflation of an air bag**

(by courtesy of Damir Isovic)

- **Schedulability condition:**
  - all jobs of all tasks must meet their deadline constraints

\[
\forall i, k, \varphi \quad BD_i \leq R_{i,k}(\varphi) \leq WD_i
\]
Background and motivation

• State-of-research [15, 27, 29, 6]
  • Best-case response time and jitter analysis of independent tasks with constrained deadlines.

• Goal
  • Best-case response time and jitter analysis of independent tasks with arbitrary deadlines.

Overview

- Background and motivation
- Real-time scheduling model
- Witnesses of non-duality
- Response-time analysis
  - Worst-case analysis
  - Best-case analysis
- Finalization jitter
- Conclusions
A classical model for fixed-priority preemptive scheduling with
- Activation jitter $AJ_i$;
- Arbitrary phasing $\varphi_i$;
- Arbitrary relative (worst-case) deadlines $WD_i$.

Note on terminology:
- Finalization time $F_{i,k} = f_{i,k} - (\varphi_i + k \cdot T_i)$;
- Response time $R_{i,k} = f_{i,k} - a_{i,k}$;
- Worst-case deadline $wd_{i,k} = a_{i,k} + WD_i$. 

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TU/e
Technische Universität
Eindhoven
University of Technology
Witnesess of non-duality – example I

<table>
<thead>
<tr>
<th>Task</th>
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<td>8.6</td>
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Worst-case response time of task $\tau_2$

**Fact:** The worst-case response time of a task $\tau_i$ with an arbitrary deadline is assumed *somewhere* in the *longest* level-$i$ active period.
Witnesses of non-duality – example I

<table>
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</table>

**Fact:** The best-case response time of a task $\tau_i$ with an arbitrary deadline is **not necessarily** assumed in the *shortest* level-$i$ active period.
Witnesses of non-duality – example I

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Best-case response time of task $\tau_2$

Fact: The best-case response time of a task $\tau_i$ with an arbitrary deadline is always assumed by the last job in a level-$i$ active period.
Witnesses of non-duality – example II

<table>
<thead>
<tr>
<th>Task</th>
<th>$T$</th>
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<th>$D$</th>
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<tbody>
<tr>
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<td>4</td>
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<td>2</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>7</td>
<td>2</td>
<td>9</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

**Fact:** When a job of a task $\tau_i$ with an arbitrary deadline assumes the best-case response time and cannot immediately start upon its activation, it does not necessarily experience interference of its previous job.
Worst-case analysis

• Based on a critical instant
  • Simultaneous release of a task with all its higher priority tasks.

• Worst-case response time
  • Longest response time in so-called level-i active period.
Worst-case response time of task $\tau_2$. 

Longest level-2 active period: $WL_2 = 35$. 
Number of activations of task $\tau_2$ in $WL_2$: $wl_2 = \lceil WL_2 / T_2 \rceil = \lceil 35/7 \rceil = 5$
Best-case analysis

• Based on an *optimal instant*
  • *Finalization* of a task at a simultaneous release of all its higher priority tasks

• Best-case response time
  • Shortest response time of the *last job* in a level-*i* active period of a length of at most *WL*<sub>*i*</sub>.

• Definition:
  • *The best-case interval* *BI*<sub>*i*</sub>(*y*) *is defined as the length of the shortest interval in which an amount of time* *y* ∈ *R*<sup>+</sup> *is available for the execution of a task* *τ*<sub>*i*</sub>.
  • *BI*<sub>*i*</sub>(*y*) *is given by* *BR*<sub>*i*</sub>(*y*) *for constrained deadlines.*
Activation of job $\tau_{2,-1}$

$BR_{2}^{(1)} = BI_{2}(C_{2}) = 6.2$

$BI_{2}(C_{2}) = 6.2$
Activation of job $\tau_{2,-2}$

\[ BR_2^{(1)} = BI_2(C_2) = 6.2 \]

\[ BR_2^{(2)} = \max_{1 \leq k \leq 2} (BI_2(k \cdot C_2) - (k - 1) \cdot T_2) = 6.2 \]

$BI_2(2 \cdot C_2) = 12.4$
Activation of job $\tau_{2,-3}$

$$BR_2^{(1)} = BI_2(C_2) = 6.2$$

$$BR_2^{(2)} = \max_{1 \leq k \leq 2} \left( BI_2(k \cdot C_2) - (k - 1) \cdot T_2 \right) = 6.2$$

$$BR_2^{(3)} = \max_{1 \leq k \leq 3} \left( BI_2(k \cdot C_2) - (k - 1) \cdot T_2 \right) = 6.6$$

$BI_2(3 \cdot C_2) = 20.6$
Activation of job $\tau_{2,-3}$

$BR_2^{(1)} = BI_2(C_2) = 6.2$

$BR_2^{(2)} = \max_{1 \leq k \leq 2} (BI_2(k \cdot C_2) - (k - 1) \cdot T_2) = 6.2$

$BR_2^{(3)} = \max_{1 \leq k \leq 3} (BI_2(k \cdot C_2) - (k - 1) \cdot T_2) = 6.6$
Activation of job $\tau_{2,-4}$

$$BR_2^{(1)} = BI_2(C_2) = 6.2$$

$$BR_2^{(2)} = \max_{1 \leq k \leq 2} (BI_2(k \cdot C_2) - (k - 1) \cdot T_2) = 6.2$$

$$BR_2^{(3)} = \max_{1 \leq k \leq 3} (BI_2(k \cdot C_2) - (k - 1) \cdot T_2) = 6.6$$

$$BR_2^{(4)} = \max_{1 \leq k \leq 4} (BI_2(k \cdot C_2) - (k - 1) \cdot T_2) = 6.6$$

$BI_2(4 \cdot C_2) = 26.8$
Activation of job $\tau_{2,-5}$

$BR_{2}^{(1)} = BI_{2}(C_{2}) = 6.2$

$BR_{2}^{(2)} = \max_{1 \leq k \leq 2} (BI_{2}(k \cdot C_{2}) - (k - 1) \cdot T_{2}) = 6.2$

$BR_{2}^{(3)} = \max_{1 \leq k \leq 3} (BI_{2}(k \cdot C_{2}) - (k - 1) \cdot T_{2}) = 6.6$

$BR_{2}^{(4)} = \max_{1 \leq k \leq 4} (BI_{2}(k \cdot C_{2}) - (k - 1) \cdot T_{2}) = 6.6$

$BR_{2}^{(5)} = \max_{1 \leq k \leq 5} (BI_{2}(k \cdot C_{2}) - (k - 1) \cdot T_{2}) = 6.6$

$BI_{2}(5 \cdot C_{2}) = 26.8$
Best-case response time of task $\tau_2$

Level-2 active period

$$BR_2 = \max_{1 \leq k \leq w_{l2}} (BI_2(k \cdot C_2) - (k - 1) \cdot T_2) = 6.6$$
Best-case analysis

- **Without activation jitter**
  - Best-case response times:
    \[ BR_i = \max_{1 \leq k \leq w_l} \left( \text{BI}_i(k \cdot \text{BC}_i) - (k - 1)T_i \right); \]
  - Best-case finalization times:
    \[ BF_i = BR_i; \]

- **With activation jitter**
  - Best-case response times:
    \[ BR_i = \max_{1 \leq k \leq w_l} \left( \text{BI}_i(k \cdot \text{BC}_i) - \begin{cases} 0 & \text{for } k = 1 \\ (k - 1)T_i + AJ_i & \text{for } k > 1 \end{cases} \right); \]
  - Best-case finalization times:
    \[ BF_i = \max_{1 \leq k \leq w_l} \left( \text{BI}_i(k \cdot \text{BC}_i) - (k - 1)T_i \right); \]
Finalization jitter – an example

<table>
<thead>
<tr>
<th>Task</th>
<th>$T$</th>
<th>$D$</th>
<th>$AJ$</th>
<th>$C$</th>
<th>$WR$</th>
<th>$BR$</th>
<th>$WF$</th>
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<td>$\tau_2$</td>
<td>5</td>
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</tr>
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</table>

**Constrained deadlines**
- $FJ_i \leq AJ_i + WR_i - BR_i$; [6]

**Arbitrary deadlines**
- $FJ_i \leq WF_i - BR_i$; [17]
- $FJ_i \leq WF_i - BF_i$; [0]

$BR_3 = 2 \Rightarrow FJ_3 = 0.6 + 8.6 - 2 = 7.2$

$BR_3 = 2 \Rightarrow FJ_3 = 8.6 - 2 = 6.6$

$BR_3 = 2.4 \Rightarrow FJ_3 = 8.6 - 2.4 = 6.2$

$FJ_3 = 8.6 - 3 = 5.4$

[0] This paper/presentation.
Contributions

• Witnesses of non-duality
• Best-case analysis for arbitrary deadlines and jitter
  • best-case response times;
  • best-case finalization times.
• Improvements:
  • finalization jitter;
  • end-to-end response times.
• Illustrated by means of examples.
• “Basis” for further extensions.
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