Improving the global schedulability analysis of overrun without payback

Uğur Keskin, Reinder J. Bril, Johan J. Lukkien
Dep. Mathematics and Computer Science
Technische Universiteit Eindhoven (TU/e)
Den Dolech 2, 5612 AZ Eindhoven, The Netherlands
u.keskin@TUE.nl

Moris Behnam, Thomas Nolte
Mälardalen Real-Time Research Centre (MRTC)
Mälardalen University
P.O. Box 883, SE-721 23 Västerås, Sweden

Abstract—Overrun without payback has been proposed as a mechanism for a stack resource policy (SRP) based synchronization for hierarchical scheduling frameworks (HSFs). In this paper we reconsider the global schedulability analysis of an HSF based on two-level fixed-priority preemptive scheduling (FPPS) using overrun without payback as a mechanism. Improved analysis is presented based on the observation that there is no need to guarantee the overrun budget before the end of the budget period, because that additional amount of resources is only meant to prevent depletion of a budget during global resource access. The resulting improvement is illustrated by means of an example. The possibility to discard the remainder of an overrun budget upon a replenishment is briefly considered as a further improvement and its potential is shown using the same example.

I. INTRODUCTION

A. Background

The Hierarchical Scheduling Framework (HSF) has been introduced to support hierarchical CPU sharing among applications under different scheduling services [1]. The HSF can generally be represented as a tree of nodes, where each node represents an application with its own scheduler for scheduling internal workloads (e.g. tasks), and resources are allocated from a parent node to its children nodes.

The HSF provides means for decomposing a complex system into well-defined parts called subsystems, which may share (so-called global) logical resources requiring mutually exclusive access. In essence, the HSF provides a mechanism for timing-predictable composition of course-grained subsystems. In the HSF, subsystems can be independently developed, analyzed and tested. Temporal isolation between subsystems is provided through budgets which are allocated to subsystems.

As large extents of embedded systems are resource constrained, a tight analysis is instrumental in a successful deployment of HSF techniques in real applications. We therefore aim at reducing potential pessimism in existing schedulability analysis for HSFs. Looking further at existing industrial real-time systems, FPPS is the de facto standard of task scheduling, hence we focus on an HSF with support for FPPS within a subsystem. Having such support will simplify migration to and integration of existing legacy applications into the HSF. Our current research efforts are directed towards the conception and realization of a two-level HSF that is based on (i) FPPS for both global scheduling of budgets (allocated to subsystems) and local scheduling of tasks (within a subsystem), (ii) the periodic resource model [1] for budgets, and (iii) the Stack Resource Policy (SRP) [2] for both inter- and intra-subsystem resource sharing. For such an HSF, we consider the overrun without payback mechanism [4], [6] that prevents depletion of a budget during global resource access by temporarily increasing the budget with a statically determined amount for the duration of that access. The term without payback means that the additional amount of budget does not have to be payed back during the next budget period. To distinguish this additional amount of budget from a normal budget, we will use the term overrun budget. In [3], it has been shown that the existing global schedulability analysis [4], [6] is pessimistic. The analytical improvement presented in that paper is based on the observation that during an overrun, as a resource is locked, not all higher priority subsystems can preempt.

In this paper, we show that the global analysis can be further improved by lifting the assumption that an overrun budget must also be guaranteed before the end of its budget period.

B. Contributions

We present improved global schedulability analysis for overrun without payback in Section IV, assuming that only the normal budget must be guaranteed before the end of the budget period. We illustrate this improved analysis and compare it with the existing analysis [4], [6] and the initial improvement presented in [3] using an example system in Section V. We briefly consider a further improvement in Section VI, i.e. the possibility to discard the remainder of an overrun budget upon a replenishment, and illustrate its potential using the same example system. We leave its analysis as future work.

II. REAL-TIME SCHEDULING MODEL

A. System model

A system $Sys$ contains a set $R$ of $M$ global logical resources $R_1, R_2, \ldots, R_M$, a set $S$ of $N$ subsystems $S_1,
\( S_1, \ldots, S_N \), a set \( \mathcal{B} \) of \( N \) budgets for which we assume a periodic resource model [1]. Each subsystem \( S_i \) has a dedicated budget associated to it. Subsystems are scheduled by means of FPPS and have fixed, unique priorities. For notational convenience, we assume that subsystems are given in order of decreasing priorities, i.e. \( S_1 \) has highest priority and \( S_N \) has lowest priority.

**B. Subsystem model**

Each subsystem \( S_i \) contains a set \( \mathcal{T}_i \) of \( n_i \) periodic tasks \( \tau_1, \tau_2, \ldots, \tau_{n_i} \) with fixed, unique priorities, which are scheduled by means of FPPS. The set \( \mathcal{R} \) denotes the subset of \( M \) global resources accessed by \( S_i \). The timing characteristics of \( S_i \) are specified by means of a triple \( \langle P_i, Q_i, X_i \rangle \), where \( P_i \in \mathbb{R}^+ \) denotes its (budget) period that serves also as deadline, \( Q_i \in \mathbb{R}^+ \) its (normal) budget, and \( X_i \) the set of maximum execution access times of \( S_i \) to global resources. The maximum value in \( X_i \) is denoted by \( X_i \).

**C. Synchronization protocol**

The overrun without payback mechanism is assumed to be used. To be able to use SRP in an HSF for synchronizing global resources (HSRP [4]), its associated ceiling terms need to be extended. For every global resource, an external resource ceiling is associated and it is defined as

\[
RC_i = \min(N, \min\{s \mid R_i \in \mathcal{R}_i\}).
\]

**D. Concluding remark**

For ease of presentation, we only consider a single global resource in the remainder of this paper, i.e. \( M = 1 \).

### III. EXISTING SCHEDULABILITY ANALYSIS

In this section, we briefly recapitulate the global schedulability analysis presented in [4] and improved global analysis described in [3]. The global schedulability analysis presented in [5], [6] looks different, but it is based on the analysis described in [4] and therefore yields the same result.

**A. Original global analysis**

The worst-case response time \( WR_i \) of subsystem \( S_i \) is given by the smallest \( x \in \mathbb{R}^+ \) satisfying

\[
x = B_i + (Q_i + X_i) + \sum_{t \leq x} \left( \frac{x}{P_i} \right) (Q_i + X_i),
\]

where \( B_i \) is the maximum blocking time of \( S_i \) by lower priority subsystems, i.e.

\[
B_i = \max(0, \max\{X_{it} \mid t > s \land X_{it} > 0, RC_i \leq s\}).
\]

Note that we use the outermost max in (3) to define \( B_i \) also in those situations where the set of values of the innermost max is empty. To calculate \( WR_i \), we can use an iterative procedure based on recurrence relationships, starting with a lower bound, e.g. \( B_i + \sum_{t \leq x} (Q_i + X_i) \). The condition for global schedulability is given by

\[
\forall 1 \leq i \leq N \quad WR_i \leq P_i.
\]

The global analysis above is similar to basic analysis for FPPS with resource sharing, where the period \( P_i \) of a subsystem \( S_i \) serves as deadline for the sum of \( Q_i \) and \( X_i \). The interference of higher priority subsystems \( S_j \) is based on the sum \( Q_j + X_j \). A superscript \( P \) will be used to refer to this basic analysis for subsystems, e.g. \( WR_i^P \).

In the sequel, we are not only interested in \( WR_i \) of \( S_i \) for particular values of \( B_i, Q_i, \) and \( X_i \), but in the value as a function of the sum of these values. We will therefore use a functional notation when needed, e.g. \( WR_i(B_i + Q_i + X_i) \).

**B. Initial improved global analysis**

The initial improved global schedulability analysis presented in [3] is based on the observation that during an overrun, as a resource is locked, not all higher priority subsystems can preempt. The analysis for this initial improvement is similar to the analysis for FPPS [7] and FPPS with preemption thresholds [8] in the sense that we have to consider all jobs in a so-called level-\( s \) active period to determine \( WR_i \) of subsystem \( S_i \). Unlike the analysis described in [7], [8], subsystems \( S_{i-1} \) till \( S_{R_C} \) cannot preempt \( S_i \) at the finalization time of \( Q_i \) when \( S_i \) is accessing \( R_i \). Similar to the original analysis [4], [6], the initial improvement takes \( P_i \) as a deadline for the total budget for \( S_i \), i.e. \( Q_i + X_i \).

### IV. DEADLINE ONLY HOLDS FOR NORMAL BUDGET

Because the overrun budget \( X_i \) is only meant to prevent depletion of a budget during global resource access, the deadline of the budget only needs to hold for the normal budget \( Q_i \) of a subsystem \( S_i \) and not for its total budget \( Q_i + X_i \). The improved analysis resulting from this observation is based on a level-\( s \) active period. The worst-case length \( WL_s \) of a level-\( s \) active period with \( s \leq N \) is given by the smallest \( x \in \mathbb{R}^+ \) that satisfies

\[
x = B_i + \sum_{t \leq x} \left( \frac{x}{P_i} \right) (Q_i + X_i).
\]

To calculate \( WL_s \), we can use an iterative procedure based on recurrence relationships, starting with a lower bound, e.g. \( B_i + \sum_{t \leq x} (Q_i + X_i) \). The maximum number \( w_l \) of jobs of \( S_i \) in a level-\( s \) active period is given by

\[
w_l = \left[ \frac{WL_s}{P_i} \right].
\]

For a job \( t_{sk} \) of \( S_i \) with \( 0 \leq k < w_l \), we split the interval from the start of the level-\( s \) active period to the finalization of job \( t_{sk} \) in two sub-intervals: a first sub-interval including the execution of the normal budget \( Q_i \) by job \( t_{sk} \) and a second sub-interval from the finalization of \( Q_i \) by \( t_{sk} \) till the finalization of \( t_{sk} \).
The worst-case finalization time $WF^Q_{sk}$ of the normal budget $Q_s$ of job $t_{sk}$ with $0 \leq k < w t_k$ relative to the start of the constituting level-$s$ active period is given by

$$WF^Q_{sk} = WR^P_s (B_s + (k + 1)Q_s + kX_s). \quad (7)$$

The worst-case response time $WR^Q_{sk}$ of job $t_{sk}$ of $Q_s$ can now be calculated as

$$WR^Q_{sk} = WR^Q_{sk} - kP_s, \quad (8)$$

and the worst case response time $WR^Q_s$ of $Q_s$ for subsystem $S_s$ is given by

$$WR^Q_s = \max_{0 \leq k < w t_s} WR^Q_{sk}. \quad (9)$$

We merely mention that this analysis is considerably simpler than initial improved analysis presented in [3].

V. RESULTS BASED ON AN EXAMPLE SYSTEM

We will now compare the original analysis, initial improved analysis, and our novel analysis presented in the previous section using an example system consisting of three subsystems with characteristics as given in Table I. Figure 1 illustrates the feasibility volumes for these three analyses.

It is clear that the initial improvement in [3] extends the feasibility volume significantly (Figure 1(b)) compared to the original analysis (Figure 1(a)). Our novel analysis only slightly extends the volume, however, as shown in Figure 1(c). To highlight the differences, let’s consider the latter two graphs in more detail. For the initial improvement, the base of the triangle-shaped plane is represented by $X_3 = 1.7$, whereas it is represented by $X_3 = 1.8$ for our novel analysis. This improvement becomes more clearly visible by considering 2-dimensional projections of the graphs as shown in Figure 2 for $X_3 = 2.0$ and $X_3 = 3.0$, i.e. our novel analysis allows $Sys_1$ to have a larger maximum value for $X_2$, extending the feasibility region.

As observed, the boundary of the feasibility volume consists of different planes. We can examine the planes and the intersections between them with the help of feasibility area ($Q_2, X_2$) graphs shown in Figure 2. For instance, if we consider the 2-d feasibility region illustrated in Figure 2(a), increasing $Q_2$ value out of region in the direction of the arrow 2 makes the system unschedulable due to $S_2$. Note that since the graph of original analysis does not have a cut off at $Q_2 = 2$, arrow 2 is not valid for it. Increasing $Q_2$ or $X_2$ above the boundary in the direction of arrow 1 makes the system unschedulable due to missing deadline of $S_3$.

![Figure 1. Feasibility volumes based on (a) original analysis, (b) initial improvements, and (c) improvement of "P3 only holds for Q1"](image)

![Figure 2. Feasibility regions of Sys1 for (a) X3 = 2.0, and (b) X3 = 3.0](image)

VI. DISCARDING $X_s$ UPON A REPLENISHMENT

The final improvement results from the observation that the remainder of the overrun budget $X_s$ of a subsystem $S_s$ can be discarded upon replenishment of its normal budget $Q_s$, because the overrun budget only serves to prevent depletion of a budget during global resource access. Figure 2 shows initial results for this improvement for two specific values of $X_3$. Apart from showing that the feasibility region is increased, it also shows that the extension is more significant for larger values of $X_3$ (Figure 2(b)).

The timeline in Figure 3 shows how the improvements affect the global schedulability of $Sys_3$ based on the simultaneous release of subsystems: $S_3$ meets its deadline (Figure 3(b)), while it is not schedulable based on the
initial improvements (Figure 3(a)). As can be observed, $S_3$ cannot complete the execution of $X_3$ within its period at time 40. After the replenishment, $S_3$ uses its main budget to complete the remaining duration of $X_3$ without any pre-emption by $S_2$. Hence, analysis based on "discarding $X_3$ upon a replenishment" makes the system schedulable allowing $S_2$ to use a larger overrun budget. Allowing $X_3$ to be discarded upon a replenishment allows the system to be schedulable for a utilization larger than 1 when the utilization is based on total budgets. Although this final improvement significantly increases the feasibility region, it is not clear yet how its analysis should be defined. We will use an example system $Sys_{II}$ consisting of two subsystems for illustration purposes; see Table II. Figure 4 shows the timeline how subsystems in $Sys_{II}$ are scheduled. The figure relates that at time 14.0, 0.4 unit of time of overrun budget is discarded upon replenishment of $S_2$. Because all pending work, in terms of remaining budgets, has completed, it may seem natural to consider time 14.0 as the end of the first level-$s$ active period. However, since $S_2$ is still using a shared resource, it effectively limits pre-emption for the duration of that resource access, i.e. till time 14.4. We may therefore have to consider an interval of length 34.6 to determine the worst-case response time $WR_3$. To obtain analysis for this improvement we need proper definitions of the notions critical instant and level-$s$ active period. Moreover, it is not clear yet how to deal with the discarded budget in the analysis. These definitions and their application for the analysis are a topic of future work.

VII. CONCLUSION

In this paper, we reconsidered the global schedulability analysis for synchronization protocols based on SRP and overrun without payback for two-level hierarchical scheduling based on FPPS. We provided improved global analysis based on the observation that the end of a period only serves as a deadline for the normal budget of a subsystem. We illustrated the resulting improvement by comparing the feasibility volumes of the original analysis and an earlier initial improved analysis with our novel analysis for an example system consisting of three subsystems. We showed that the analysis can be further improved based on the observation that the remainder of the overrun budget can be discarded at a replenishment. The initial results for this latter improvement look promising. The analysis for this improvement is considered future work.

ACKNOWLEDGEMENTS

We thank Martijn M.H.P. van den Heuvel and Mike Holenderski for discussions and comments.

REFERENCES