Cortona School in Neurogeometry 2017,

Organizers
D. Alekseevsky (Russian Academy of Sciences),
G. Citti (Univeristy of Bologna)
J. Petitot (CAMS-EHESS)
A. Sarti (CAMS-EHESS)

SCOPE
Scope of the school is to provide students at the end of Master classes or at the beginning of PhD school with an introduction to Neurogeometry. Instruments of differential geometry in anisotropic structures and experimental brain data allow to build efficient models of the visual cortex, with applications to image processing.

SCHEDULE LECTURES BY DUITS

Title: ***Orientation Score Theory and its Solutions to Cortical PDE, ODE and Wavelet Models.***

Speaker:
Remco Duits,
Department of Mathematics & Computer Science, Eindhoven University of Technology.

Second week 10-July 2017 until 14-July 2017

	M	T	W	T	F
9.30				Barbieri	
10.00		Barbieri	Barbieri		Barbieri
10.30				Duits	
11.00		coffee break	coffee break	Lecture 3	coffee break
11.30	Barbieri	Duits	Duits	coffee break	Duits
12.00		Lecture1	Lecture 2	Duits	Lecture 5
12.30				Lecture 4	Closing
13.00	Lunch	Lunch	Lunch	Lunch	
14.00					
1500	Students work	Students work	Students work	Students work	

NB. In the lecture notes of Lecture 1 and Lecture 4, there will be links to a few *Mathematica* exercises to http://www.lieanalysis.nl (select education button)
For hands-on experience:
 If you have *Mathematica 11.0* (Wolfram Research) installed you can do the exercises, and play with the code. If not, you can look at http://www.lieanalysis.nl/201-2/
with html renders of the notebooks with output instead to get an impression.

Lecture 1 (invertible multi-orientation image representations)
Introduction to Image Processing via Invertible Orientation Scores.

Content: In this 1st lecture we aim for introduction and motivation of the orientation score framework:

      - What is an invertible orientation score?
      - Why do we need invertible orientation scores?
      - Why must we apply left-invariant processing on them?
      - What medical image analysis applications can we tackle with them?

Exercise Topics:

      - Condition numbers of the transformation.
      - Proofs of general unitarity results. Reproducing kernel Hilbert spaces.
      - the Lie group structure on the domain of an orientation score.

Course Material Lecture 1:
Lecture notes will be provided digitally.

It is based on our scientific works:
http://bmia.bmt.tue.nl/people/RDuits/IJCV2007.pdf
http://erikbekkers.bitbucket.io/data/pdf/Bekkers_PhD_Thesis.pdf   (2016, section 2 only)
http://bmia.bmt.tue.nl/people/RDuits/THESISRDUITS.pdf   (2005, section 4 only)
http://bmia.bmt.tue.nl/people/RDuits/qampartI.pdf (2010, section 1 & 2 only)

Lecture 2 (design of multi-orientation image representations)
Design of Invertible Orientation Scores of 2D and 3D images.

Content: In this 2nd lecture we focus on the following topics:

      - Design of 2D Cake-wavelets.
      - Design of 3D Cake-wavelets.
      - Definition proper wavelet and proper wavelet sets.
      - Design via of proper wavelets via Harmonic oscillator
      - Design of proper wavelets via gen. Zernike wavelets
      - Relating the forward orientation score transform of 2D images to inverse Fourier transform on SE(2).
      - References to Tutorials of the Lie Analysis Mathematica 11 Package.

Exercise Topics:

      - Design of cake-wavelets in Fourier domain
      - Proper wavelets.
      - Questions on playing with cake-wavelet design in our Lie Analysis package.

Course material Lecture 1:
Lecture notes will be provided digitally.

NB. It is based on our scientific works:
http://bmia.bmt.tue.nl/people/RDuits/qampartI.pdf (2010, appendix A only)
http://bmia.bmt.tue.nl/people/RDuits/InvertibleOrientationScores.pdf (2015)
http://bmia.bmt.tue.nl/people/RDuits/InvertibleOrientationScoresof3DimagesComplete.pdf (2017)
http://bmia.bmt.tue.nl/people/RDuits/rana13-17.pdf (2015)
and
Invertible Orientation Scores of 3D images cf. arxiv, Janssen, Janssen, Bekkers, Olivan Bescos & Duits (2017)

Lecture 3 (solutions to cortical models for contour completion and enhancement)
Exact Solutions to Linear Left-Invariant PDE's on SE(2) and on SE(3).

Content: In this 3rd lecture we focus on the following topics:
- Exact solutions to Linear left-invariant PDE's on SE(2).
- Exact solutions to Hypo-elliptic Diffusion on SE(3) .
- Exact solutions to Mumford's direction Process on SE(3).

Exercise Topics:
-A review on well-known (and related) exact solutions for diffusions and Tikhonov regularization on a sphere.
-Method of separation exercises tackling specific sub-problems on SE(d).

Course Material Lecture 3:
We will use the articles
http://bmia.bmt.tue.nl/people/RDuits/SE3JorgPortegiesDuits.pdf (to appear soon in DGA 2017)
http://bmia.bmt.tue.nl/people/RDuits/Final_1411nm.pdf (chapter 5 ONLY, 2016)

NB.
Other related refs:
http://bmia.bmt.tue.nl/people/RDuits/qampartI.pdf (2010)
http://bmia.bmt.tue.nl/people/RDuits/Duits-VanAlmsick.pdf (2008)

http://bmia.bmt.tue.nl/people/RDuits/SE3JorgPortegiesDuits.pdf (to appear soon in DGA 2017)
http://bmia.bmt.tue.nl/people/RDuits/SSVMNorwaySummarySE2.pdf (2009)
The intrinsic hypoelliptic Laplacian and its heat kernel on unimodular Lie groups
(Agrachev, Boscain, Gauthier, Rossi, 2009)
http://www.win.tue.nl/analysis/reports/rana05-43.pdf (2005)

Lecture 4 (tracking via `shortest curves' in orientation scores)
Globally Optimal Sub-Riemannian geodesics in the projective line bundle.
Improvements via Asymmetric Finsler geometry: key-points instead of cusps !

Content: In this 5th lecture we illustrate geodesic tracking in orientation scores:
- Sub-Riemannian Geodesics in SE(2) and in the projective line bundle, and vessel-tracking applications.
- Sub-Riemannian Geodesics in SE(3) and DW-MRI applications.
- The problem of cusps in spatial projections of SR geodesics.

- The solution to the problem of cusps: Turn off the reverse gear of the Reeds-Shepp car.

Exercise Topics:
- Derive the Hamiltonian equations for sub-Riemannian geodesics in SE(2).
- Preservation laws and symmetries of sub-Riemannian geodesics in SE(d), d=2,3.
- Analyze the shape of the sub-Riemannian spheres on SE(2) and their intersection with the 1st Maxwell set.
- Analyze the shape of the sub-Riemannian spheres on the projective line bundle and their intersection with the 1st Maxwell set.
- Apply a vessel-tracking in a challenging industrial medical imaging example in a prepared Mathematica 11 notebook.

Course Material Lecture 4:
Lecture Notes will be provided digitally.

It is based on the following scientific works:
https://arxiv.org/pdf/1612.06137, (2017, Duits, Meesters, Mirebeau, Portegies)
http://www.sciencedirect.com/science/article/pii/S092842570300072X (2003, Petitot: SR-cortical models)
http://link.springer.com/article/10.1007/s10851-005-3630-2 (2006, Citti-Sarti: SE(2)-SR-Geodesic cortical model)

ftp://ftp.botik.ru/rented/CPRC/www/sachkov/max_sre2_COCV.pdf (2009, Sachkov-Moiseev: SE(2)-SR-Geodesic solutions and Maxwell sets)

http://www.numdam.org/item/COCV_2011__17_2_293_0 (2011, Sachkov: Optimal synthesis and cut locus SE(2)-SR-Problem)

http://www.lsis.org/rossif/articles/Suzdal08.pdf (2010, Boscain, Charlot & Rossi: the problem of cusps)
http://bmia.bmt.tue.nl/people/RDuits/cusp.pdf (2014, SE(2)-SR-Geodesic model: Analysis of cusps in spatial projections SR-geodesics and geometric properties)

http://bmia.bmt.tue.nl/people/RDuits/siims.pdf (2015, SE(2)-SR-Geodesic model: Data-driven global minimizers)

http://bmia.bmt.tue.nl/people/RDuits/Paper1_81_REMCO.pdf (2016, SE(3)-SR-Geodesics solutions)
http://bmia.bmt.tue.nl/people/RDuits/GSI2017.pdf (2017, SR-geodesics and 1st Maxwell sets on projective line bundle)

http://bmia.bmt.tue.nl/people/RDuits/main_FM_full.pdf (2017, full survey minimizing geodesics in position and orientation space & solution to problem of cusps)

Lecture 5 (crossing-preserving diffusion via `straight curves' in orientation scores)
Crossing-preserving diffusions via locally adaptive frames in invertible orientation scores.

Content: In this 4th lecture we focus on the following topics:
- Exponential Curve fits of order 1.
- Exponential Curve fits of order 2.
- Crossing-Preserving Diffusions on SE(2).
- Crossing-Preserving Diffusions on SE(3).
- The minus Cartan connection: "Straight curves" vs. "shortest curves".

Course Material Lecture 5:
We will use the article:
http://bmia.bmt.tue.nl/people/RDuits/1502.08002v3.pdf (2016)

NB. Further ref:
and http://erikbekkers.bitbucket.io/data/pdf/Bekkers_PhD_Thesis.pdf (2016, ch:4.5, ch:11)