International Workshop on Geometry, PDE's and Lie Groups in Image Analysis

24-26 August 2016 – Eindhoven

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Abstract

The synergy and interaction between the mathematical fields of Geometric Control, Partial Differential Equations, Lie group Analysis, Harmonic Analysis, Variational Methods and the applied fields of Image Analysis, Numerical Analysis, Neurogeometry and Neuroimaging is developing rather strongly over the past few decades, and has attracted many researchers. This international workshop aims to further facilitate the fruitful interaction and consists of 3 sessions:

A: Geometry and Control in Vision (chair: Yuri Sachkov)
B: Lie Group Analysis and PDEs in Image Analysis (chairs: Remco Duits & Erik Bekkers)
C: Differential Geometry in Neuro-imaging (chair: Andrea Fuster)

The workshop is part of the ERC-Lie Analysis project of Remco Duits.

Wifi: On TU/e campus (including "de Zwarte Doos") there is eduroam available and a TU/e guest network (SSID: TUe-guest).

Session A: Geometry and Control in Vision

Wednesday 24th August 2016
Chair: ***Yuri Sachkov***

A1: Geodesics, Laplacians and random walks in sub-Riemannian geometries

Ugo Boscain *(Ecole Polytechnique CMAP, Paris)*

In this talk I will connect via the random walks point of view, the geodesics and the Laplacian in a sub-Riemannian manifold M. This problem is not trivial even in the Riemannian context and passes through the definition of a volume. We define two type of Laplacians: the macroscopic one as the divergence of the horizontal gradient once a volume in the ambient space is fixed and the microscopic one as the operator associated with a geodesic random walk. This second definition requires to fix a probability density on the cylinder of initial contitions of covectors in T^*M_q, where q is the starting point. This cylinder parametrizes the different geodesics. We study under which conditions these two operator coincide. We will see that the result strongly depend on the type of sub-Riemannian structure. The main purpose is to understand which is the most natural diffusion equation that should be used in an algorithm of image reconstruction.

A2: Spatio-temporal geometry of motion perception in primary visual cortex

Davide Barbieri *(Dep. de Matemticas, Universidad Autnoma de Madrid)*

The classical linear spatio-temporal behavior of V1 receptive fields can be characterized as three dimensional Gabor filters, selective to locally oriented uniformly moving stimuli, whose distribution minimizes the uncertainty in the estimation of wavefronts normal velocity [1].

This family of filters map moving visual stimuli onto a five dimensional manifold $\mathcal{M}$ of spatial positions and activation times, together with the local features of orientations and velocities. The trajectories of contours of moving stimuli in the visual space are lifted to two dimensional legendrian submanifolds in the contact structure of $\mathcal{M}$. A signal propagation process can thus be defined as a diffusion on the feature variables with drift on the spatio-temporal variables, performed along the admissible directions of such a contact structure. The fundamental solution of the associated density equation can be used to model a spatio-temporal connectivity in primary visual cortex and feed a neural field equation on the feature space $\mathcal{M}$. Such a dynamics is actually able to reproduce neurophysiological findings on cortical responses of motion detection [2].

Such connectivity kernels may also be used as a measure of distance between two points in terms of the length of an admissible spatio-temporal trajectory that can connect them. This geometric representation provides directed graph whose vertices are liftings on $\mathcal{M}$ of points in moving stimuli and whose edges are given by the obtained distance. By a spectral analysis of such a graph, one can obtain a partition which clusters the best accessed areas in terms of the introduced geometric diffusion. The result is the emergence of perceptual units associated to contours in motion [3].

[1] G. Cocci, D. Barbieri, A. Sarti. Spatio-temporal receptive fields of cells in V1 are optimally shaped for stimulus velocity estimation. JOSA A (2012).

[2] D. Barbieri, G. Citti, G. Cocci, A. Sarti. A cortical-inspired geometry for contour perception and motion integration. J. Math. Imaging Vision (2014).

[3] G. Cocci, D. Barbieri, G. Citti, A. Sarti. Cortical spatio-temporal dimensionality reduction for visual grouping. Neural Comput. (2015).

A3: Tracking of Lines in Spherical Images via Sub-Riemannian Geodesics in SO(3)

***Alexey Mashtakov** (Control Processes Research Center, Program Systems Institute of RAS, Moscow)*

In some imaging applications (e.g. in retinal imaging) it is natural to model the data by spherical images. In order to detect salient lines in these images we consider the problem of minimizing the functional $\int_0^l \mathfrak{C}(\gamma(s))\sqrt{\xi^2 + k_g^2(s)}\,\mathrm{d}s$ for a curve γ on a sphere with fixed boundary points and directions. The total length l is free, s denotes the spherical arclength, and k_g denotes the geodesic curvature of γ. Here the analytic external cost $\mathfrak{C} \geq \delta > 0$ is obtained from spherical data. We lift this problem to the sub-Riemannian (SR) problem on Lie group $SO(3)$, and show that the spherical projection of certain SR-geodesics provides a solution to our curve optimization problem. In fact, this holds only for the geodesics whose spherical projection does not exhibit a cusp. The problem is a spherical extension of a well-known contour perception model, where we extend the model by U. Boscain and F. Rossi to the general case $\xi > 0$, $\mathfrak{C} \neq 1$. For $\mathfrak{C} = 1$ we derive SR-geodesics and evaluate the first cusp time. We show that these curves have a simpler expression when they are parameterized by spherical arclength rather than by sub-Riemannian arclength. The case $\mathfrak{C} \neq 1$ (data-driven SR-geodesics) we solve via a SR Fast Marching method. Finally we show an experiment of vessel tracking in a spherical image of the retina, and study the effect of including the spherical geometry in analysis of vessels curvature.

A4: Some New Variational Methods for Medical Image Segmentation

***Xiaoping Yang** (Dep. of Mathematics, Nanjing University China)*

Segmentation plays a very important role in medical image process. Suffering from low contrast, weak or missing boundaries and inhomogeneity, Segmentation of medical images becomes a challenging work. In this talk, we focus on our new variational segmentation models based on boundary descriptors, isoperimetric inequality and shape prior, respectivey. We also show the effciency of our models through some numerical experiments and comparisons.

A5: Barycentric Subspace Analysis: an extension of PCA to Manifolds

***Xavier Pennec** (INRIA Sophia-Antipolis, France)*

I address in this talk the generalization of Principal Component Analysis (PCA) to Riemannian manifolds and potentially more general stratified spaces. Tangent PCA is often sufficient for analyzing data which are sufficiently centered around a central value (unimodal or Gaussian-like data), but fails for multimodal or large support distributions (e.g. uniform on close compact subspaces). Instead of a covariance matrix analysis, Principal Geodesic Analysis (PGA) and Geodesic PCA (GPCA) are proposing to minimize the distance to Geodesic Subspaces (GS) which are spanned by the geodesics going through a point with tangent vector is a restricted linear sub-space of the tangent space. Other methods like Principal Nested Spheres (PNS) restrict to simpler manifolds but emphasize on the need for the nestedness of the resulting principal subspaces.

In this work, we first propose a new and more general type of family of subspaces in manifolds that we call barycentric subspaces. They are implicitly defined as the locus of points which are weighted means of $k + 1$ reference points. As this definition relies on points and do not on tangent vectors, it can also be extended to geodesic spaces which are not Riemannian. For instance, in stratified spaces, it naturally allows to have principal subspaces that span over several strata, which is not the case with PGA. We show that barycentric subspaces locally define a submanifold of dimension k which generalizes geodesic subspaces. Like PGA, barycentric subspaces can naturally be nested, which allow the construction of inductive forward nested subspaces approximating data points which contains the Frechet mean. However, it also allows the construction of backward flags which may not contain the mean. Second, we rephrase PCA in Euclidean spaces as an optimization on flags of linear subspaces (a hierarchies of properly embedded linear subspaces of increasing dimension). We propose for that an extension of the unexplained variance criterion that generalizes nicely to flags of barycentric subspaces in Riemannian manifolds. This results into a particularly appealing generalization of PCA on manifolds, that we call Barycentric Subspaces Analysis (BSA).

* Xavier Pennec. Barycentric Subspaces and Affine Spans in Manifolds Geometric Science of Information GSI'2015, Oct 2015, Palaiseau, France. Proceedings of Geometric Science of Information GSI'2015.

* Xavier Pennec. Barycentric Subspaces Analysis on Spheres. Mathematical Foundations of Computational Anatomy (MFCA'15), Oct 2015, Munich, Germany. pp.71-82, 2015, Proceedings of the fifth international workshop on Mathematical Foundations of Computational Anatomy (MFCA'15).

A6: The problem of stability in early vision, conformal geometry and spherical model of hypercolumns by Bressloff-Cowan

Dmitri Alexeevsky *(Institute for Information Transmission Problems RAS, Moscow)*

As an optical devise, an eye projects objects of external world (e.g. a curve on a screen) onto the retina by means of the central projection. Even when the gaze is fixed, the eye is participates in different types of movements: tremor, drift and saccades , and the image of the object on retina permanently changes. However, unmoved external object are perceived by a brain as unmoved in spite of the movement of its image on retina. The problem of stability consists in description of mechanism of compensation of such image transformations on retina caused by the eye rotations.

We will show that under some assumptions, the changes of the image on retina is described by a conformal transformations and the problem of stability can be formulated as the classical problem of conformal geometry of the 2-sphere: description of a curve on the conformal sphere up to conformal transformations (Conformal Frenet Problem).

We will discuss the spherical model of hypercolumns in primary visual cortex VI, proposed by Bressloff and Cowan in 2002 and show how some modification of this model, inspired by contact Petitot model of cortex VI and its symplectic generalization, proposed by Petitot-Citti-Sarti can be applied to stability problem.

A7: Exact Solutions and Analysis of the 3D direction Process & (Hypo-)elliptic Diffusion on SE(3)

Jorg Portegies *(CASA, TU/e Eindhoven)*

In a recent paper [1] we have introduced an exact expression for the solution kernel for Diffusion and Convection-Diffusion on the roto-translation group SE(3). This includes the elliptic and the hypo-elliptic case, and is a 3D extension of (part of) earlier results in the SE(2)-case, [2,3,4]. This goes via application of the Fourier transform in the spatial variables, and spectral decomposition of the resulting differential operator. Alternative expressions via the Fourier transform on SE(3) will also be presented, connecting to the general framework in [4]. Furthermore, we present a numerical approach that is closely related to this exact solution. In particular the hypo-elliptic diffusion is very useful for enhancement of diffusion magnetic resonance imaging (dMRI) data. We provide some examples in dMRI that show the benefit of this type of regularization [5].

[1] Portegies & Duits. New Exact and Numerical Solutions of the (Convection-)Diffusion Kernels on SE(3), submitted to ACHA. arXiv:1604.03843 [math], 2016.

[2] Duits & van Almsick. The Explicit Solutions of Linear Second Order Stochastic Evolutions on SE(2), QAM-AMS, 2008.

[3] Duits & Franken. Line enhancement and completion via left-invariant scale spaces on SE(2) , LNCS, 2009.

[4] Agrachev, Boscain, Gautier & Rossi. The intrinsic hypoelliptic Laplacian and its heat kernel on unimodular Lie groups, Journ. of Func. An., 2009.

[5] Portegies, Fick, Sanguinetti, Meesters, Girard, Duits. Improving Fiber Alignment in HARDI by Combining Contextual PDE Flow with Constrained Spherical Deconvolution, PLoS ONE, 2015.

Session B: Lie Group Analysis and PDEs in Image Analysis

Thursday 25th August 2016
Chairs: ***Remco Duits & Erik Bekkers***

B1: Connecting Linear Systems and Morphology

Joachim Weickert *(Mathematical Image Analysis Group, Saarland University)*

It is well-known that there are striking analogies between linear shift-invariant systems and morphological systems for image analysis. So far, however, the relations between both system theories are mainly understood on a pure convolution / erosion level. A formal connection on the level of differential or pseudodifferential equations and their induced scale-spaces is still missing. The goal of this talk is to close this gap. We present a simple and fairly general dictionary that allows to translate any linear shift-invariant evolution equation into its morphological counterpart and vice versa. It is based on a scale-space representation by means of the symbol of its (pseudo)differential operator. Introducing a novel transformation, the Cramér–Fourier transform, puts us in a position to relate the symbol to the structuring function of a morphological scale-space of Hamilton–Jacobi type. As an application of our general theory, we derive the morphological counterparts of many linear shift-invariant scale-spaces, such as the Poisson scale-space, alpha-scale-spaces, summed alpha-scale-spaces, relativistic scale-spaces, and their anisotropic variants. Our findings are illustrated by experiments.

Joint work with **Martin Schmidt**.

B2: Seeking invariants in the image to surface (inverse) map

Steven Zucker *(Computer Science and Biomedical Engineering, Yale University) and* ***Ben Kunsberg*** *(Brown University)*

Shading and contour drawings are two sources of monocular image information from which shape can be inferred. Both inferences are ill-posed in the classical sense, in that the map between images and surfaces is many-to-many. Researchers in computer vision and visual psychophysics normally consider these problems to be different from one another, although in both cases they seek to identify sufficient 'priors' to make the problem well-posed. We propose a radically different approach by analyzing what is common between them. Instead of seeking the 'unique' surface suggested by a given image, we focus on key parts of each of them. The map we seek is over (a representation of) these key parts. Representing shading information geometrically, we start with the isophotes and their associated gradient flow. The key parts are defined via a novel limiting procedure, by which we show how certain shading configurations converge to a well-defined line drawing. Our main result establishes a correspondence between these line drawings and the Morse-Smale complex, a topological framework for gradient flows, defined on the surface. The result holds for a general class of rendering functions, and explains why different people see qualitatively similar but quantitatively different surfaces when viewing a given image.

B3: A cortically based system of PDEs in Lie groups for modal completion

Giovanna Citti *(Dep. of Mathematics, University of Bologna)*

We present a model of modal completion expressed in terms of PDE in Lie groups, joint work with **A. Sarti**. Neurogeometrical methods have been deeply applied to models of the visual cortex and face a modal completion problems. Here we study the combined action of two cortical areas, and the primary cortex V1. They are modelled with two different Lie groups, inspired respectively to the retinex equation and the neurogeometrical models. A Lagrangian functional is proposed, invariant with respect to the Lie group structures. The Euler-Lagrange equations naturally lead to an invariant system of PDE. As a result we can solve the problem of modal completion, and apply it to many significant figures, as the Kanizsa triangle.

B4: Time-causal and time-recursive spatio-temporal receptive fields for computer vision and computational modelling of biological vision

Tony Lindeberg *(Computational Brain Science Lab, KTH Stockholm)*

When operating on time-dependent image information in real time, a fundamental constraint originates from the fact that image operations must be both time-causal and time-recursive.

In this talk, we will present an improved model and theory for time-causal and time-recursive spatio-temporal receptive fields, obtained by a combination of Gaussian filters over the spatial domain and first-order integrators or equivalently truncated exponential filters coupled in cascade over the temporal domain. This receptive field family obeys scale-space axiomatics in the sense of non-enhancement of local extrema over the spatial domain and non-creation of new local extrema over time for any purely temporal signal and does in these respects guarantee theoretically well-founded treatment of spatio-temporal image structures at different spatial and temporal scales.

By a logarithmic distribution of the temporal scale levels in combination with the construction of a time-causal limit kernel based on an infinitely dense distribution of the temporal scale levels towards zero temporal scale, it will be shown that this family allows for temporal scale invariance although the temporal scale levels by the theory have to be discrete. Additionally, the family obeys basic invariance or covariance properties under other classes of natural image transformations including spatial scaling transformations, rotations/affine image deformations over the spatial domain, Galilean transformations of space time and local multiplicative intensity transformations. Thereby, this receptive field family allows for the formulation of multi-scale differential geometric image features with invariance or covariance properties under basic classes of natural image transformations over space-time.

It is shown how this spatio-temporal scale-space concept (i) allows for efficient computation of different types of spatio-temporal features for purposes in computer vision and (ii) leads to predictions about biological receptive fields with good qualitative similarities to the results of cell recordings in the lateral geniculate nucleus (LGN) and the primary visual cortex (V1) in biological vision.

T. Lindeberg (2015) Time-causal and time-recursive spatio-temporal receptive fields, Journal of Mathematical Imaging and Vision, in press. Preprint at arXiv:1504.02648.

T. Lindeberg (2013) A computational theory of visual receptive fields, Biological Cybernetics, 107(6):589635.

T. Lindeberg (2013) Invariance of visual operations at the level of receptive fields, PLOS One, 8(7):e66990.

T. Lindeberg (2011) Generalized Gaussian scale-space axiomatics comprising linear scale space, affine scale space and spatio-temporal scale space, Journal of Mathematical Imaging and Vision, 40(1):3681.

B5: Computation of Curvature Penalized Shortest Paths via the Fast Marching Algorithm

Jean-Marie Mirebeau *(Lab. de mathematiques d'Orsay, University Paris-Sud CNRS)*

Motivated by applications to motion planning and image segmentation, we consider shortest paths models with a curvature penalization, such as Euler/Mumford elasticas, or the Reed-Shepp car with or without rear gear. Our numerical strategy, for computing the path of minimal energy joining two given points, involves approximating these singular models via strongly anisotropic Riemannian or Finslerian metrics, on the product space $R^d \times S^{d-1}$. The associated eikonal equations are then solved using specialized variants of the Fast-Marching algorithm.

B6: Lie-groups and diffusion equations on image feature representations motivated by computational neuroscience

Michael Felsberg *(Institutionen for systemteknik, Linkoping University)*

Visual feature descriptors are essential elements in most computer and robot vision systems. Also, the human brain represents sensorimotor information at a suitable abstraction level through varying activation of neuron populations. In previous work, computational models have been derived that agree with findings of neurophysiological experiments on the representation of visual information by decoding the underlying signals. However, the represented variables have a bias toward centers or boundaries of the tuning curves. Despite the fact that feature descriptors in computer vision are motivated from neuroscience, the respective decoding methods have been derived largely independent. From Lie-group constraints, we derive decoding schemes for biologically motivated feature descriptors with a minimum amount of redundancy and suitable invariance properties. Based on the resulting algebraic constraints, we show formally how the decoding problem is formulated as an unbiased maximum likelihood estimator and we derive a recurrent inverse diffusion scheme to infer the dominating mode of the underlying distribution. Joint work with **Kristoffer fjll** and **Reiner Lenz**.

Felsberg Michael, fjll Kristoffer, Lenz Reiner. Unbiased Decoding of Biologically Motivated Visual Feature Descriptors. Frontiers in Robotics and AI (2015)

B7: A variational formulation of the sub-Riemannian model of the primary visual cortex

Dario Prandi *(CEREMADE, University Paris-Dauphine)*

In this talk we present a new mathematical model for the human primary visual cortex V1 and its applications to image processing, based on the well-known sub-Riemannian Citti-Petitot-Sarti model. In this model the primary visual cortex is represented as the bundle $PT\mathbb{R}^2$ of directions of the plane, where each point corresponds to a neuron with both spatial location and local orientation preferences. This structure is then endowed with a sub-Riemannian metric, mimicking the long and short range connections between neurons.

The main novelty of ou approach is the definition of the lift of 2D images to this space, and more generally of image restoration tasks (e.g. denoising or inpainting), through a dissipation of the energy by the cortical structure. More precisely, we define a variational procedure that lifts a visual stimulus (i.e. an image) to the cortical state that both minimizes its sub-Riemannian H^1 norm (minimum effort of the cortex hypothesis) and at the same time re-projects back to the same image (consistency hypothesis). In sharp contrast with previous definitions of lifts (e.g. via Gabor wavelets), this method does not make use of an a-priori fixed lift procedure. This allows us to integrate into this procedure the sub-Riemannian structure of the cortex, which in turn, makes the lift sensitive to the curvature of objects composing the image, without resorting to any post-processing diffusion over the cortical surface.

We will focus, in particular, on the characterization of this lift as a wavelet-type transform lift where the wavelet is a highly anisotropic distribution on $\mathbb{R}^2$, which allows for a rigorous derivation of the so-called extra-classical receptive fields.

This is joint work with **G. Peyré**, **J.-M. Mirebeau** and **A. Sarti**.

B8: Image processing via invertible orientation scores of 3D images

Michiel Janssen *(CASA, TU/e Eindhoven)*

Locally adaptive differential frames are used in differential invariants and PDE-flows on d-dimensional images. However, at complex structures, these frames are not well-defined. Therefore, we propose locally adaptive frames on invertible orientation scores defined on the coupled space of positions and orientations given by the Lie group quotient SE(d) / (0 x SO(d-1)), d=2,3. This allows for multiple well-defined frames per position, one for each orientation. We compute these frames via first order/ second order local exponential curve fits to the data. We will also show that the underlying left Cartan connection is indeed the correct choice of connection in the (locally adaptive) processing of orientation scores. Applications include improved crossing-preserving vesselness filtering, vessel segmentations and diffusions.

Joint work with **R. Duits**.

B9: Multiscale Segmentation via Bregman Distances and Nonlinear Spectral Analysis

Christophe Brune *(Dep. of Applied Mathematics, University of Twente)*

In biomedical imaging reliable segmentation of objects (e.g. from small cells up to large organs) is of fundamental importance for automated medical diagnosis. New approaches for multi-scale segmentation can considerably improve performance in case of natural variations in intensity, size and shape. This talk aims at segmenting objects of interest based on shape contours and automatically finding multiple objects with different scales. The overall strategy is to combine nonlinear segmentation with scales spaces and spectral decompositions recently introduced in literature. For this we generalize a variational segmentation model based on total variation using Bregman distances to construct an inverse scale space. This offers the new model to be accomplished by a scale analysis approach based on a spectral decomposition of the total variation. As a result we obtain a very efficient, (nearly) parameter-free multiscale segmentation method that comes with an adaptive regularization parameter choice. The added benefit of our method is demonstrated by systematic synthetic tests and its usage in a new biomedical toolbox for identifying and classifying circulating tumor cells. Due to the nature of nonlinear diffusion underlying, the mathematical concepts in this work offer promising extensions to nonlocal classification problems.

Session C: Differential Geometry in Neuro-imaging

Friday 26th August 2016
Chair: ***Andrea Fuster***

C1: Non-holonomicity and Parellel Transport in Biological Fibrous Composites

Kaleem Siddiqi *(School of computer science, McGill University)*

What does a Stomatopods dactyl club, an insects cuticle, an egg shell and the mammalian heart wall have in common? Each is a fibrous composite that must withstand repetitive high energy loading events while being tolerant to damage. The average human heart, for example, will beat more than 2.5 billion times in the course of a lifespan. But mechanical strength is not the only consideration for proper biological function. Dynamic structures such as the myocardium must also facilitate electrical conduction while optimizing ejection fraction. Using minimal surface theory and Diffusion Magnetic Resonance Imaging, I will show that a scale that goes beyond that of a single cell nature has found an elegant solution to such problems, which is rooted in a notion of parallel transport.

http://www.cim.mcgill.ca/ shape/

C2: Bundle Analytics: Computation in the Space of Streamlines

Maxime Descoteaux *(Sherbrooke Connectivity Imaging Lab, University of Sherbrooke)*

In this talk, I will introduce the concept of bundle analytics. As of today, there exist very little computational tools to process tractograms, fiber bundles or streamlines, in their specific space, i.e. the space of streamlines. Just as surface meshes can improve on the precision of registration and segmentation in the image-world for T1 images for example, streamlines and their representations can lead to new powerful techniques for tractography analysis. I will show examples of fiber bundle clustering, automatic bundle recognition/segmentation, outlier rejection and atlas construction based on bundle analytics.

C3: Whole-brain cortical parcellation: A hierarchical method based on dMRI tractography

David Moreno *(Mint Labs, Barcelona)*

In modern neuroscience there is general agreement that brain function relies on networks and that connectivity is therefore of paramount importance for brain function. Accordingly, the delineation of functional brain areas on the basis of diffusion magnetic resonance imaging (dMRI) and tractography may lead to highly relevant brain maps. Existing methods typically aim to find a predefined number of areas and/or are limited to small regions of grey matter. However, it is in general not likely that a single parcellation dividing the brain into a finite number of areas is an adequate representation of the function-anatomical organization of the brain. In this work, we propose hierarchical clustering as a solution to overcome these limitations and achieve whole-brain parcellation. We demonstrate that this method encodes the information of the underlying structure at all granularity levels in a hierarchical tree or dendrogram. We develop an optimal tree building and processing pipeline that reduces the complexity of the tree with minimal information loss. We show how these trees can be used to compare the similarity structure of different subjects or recordings and how to extract parcellations from them. This approach yields a more exhaustive representation of the real underlying structure and successfully tackles the challenge of whole-brain parcellation.

C4: PDEs for Quantitative Biomedical Image Analysis

Thomas Schultz *(Visualization and Medical Image Analysis Group, University of Bonn)*

In this talk, two specific PDEs and their application to quantitative biomedical image analysis will be discussed.

In the first part, a novel multi-scale anisotropic fourth-order diffusion filter will be presented that makes use of the curvature enhancing properties of fourth-order diffusion to more precisely localize coherent ridge and valley structures, such as vessels or retinal layers in medical images. The novel PDE and its efficient implementation will be discussed. A direct comparison to a range of previously proposed vessel-enhancing filters indicates an improved agreement with ground truth or manual segmentations, respectively, in simulated and real data from ophthalmology.

In the second part, I will demonstrate how a previously known relationship between the scale of image structures and changes in image intensity in Total Variation Flow can be used to quantify the scale and density of spots in Stimulated Emission Depletion (STED) microscopy without requiring their individual segmentation.

C5: Implementation of Diffusions on SE(3) in DIPY and Applications in DW-MRI: Localization of the Optic Radiation for Epilepsy Surgery

Stephan Meesters *(CASA TU/e & Epilepsy Institute Kempenhaeghe Heeze)*

In this talk the following topics are discussed: 1) Processing of HARDI data using contextual PDEs. We demonstrate its potential in crossing-preserving enhancement of ODF/FOD fields where the aim is to enhance the alignment of elongated structures such that crossings are maintained; 2) Cleaning the output of tractography using the fiber to bundle coherence (FBC) measures. FBC provides us with a quantitative measure of fiber alignment and is therefore useful in pruning the results of tractography algorithms by removing spurious fibers that are identified by a low FBC; 3) Application: the optic radiation is reconstructed with probabilistic tractography in support of temporal lobe resective surgery. Parameter estimation is performed to regulate the removal of spurious streamlines near the Meyers loop based on the FBC measures, while preserving the anatomically plausible streamlines. Standardized estimation of this parameter is achieved using a test-retest procedure based on the stability of the distance from the Meyers loop (ML) to the temporal pole (TP).

C6: Cartan connections on Lie groups with applications to neuroimaging (Alzheimer's Disease modeling)

Marco Lorenzi *(Translational Imaging Group, University College London)*

In this talk I will review the theoretical foundations of non-linear image registration parameterized by stationary velocity fields (SVF), and the application of this powerful framework to the analysis of brain changes in Alzheimers disease.

I will first show that the group of diffeomorphisms parameterised by SVF has an affine manifold structure based on the canonical Cartan connections. This enables to explicitly define geodesics and the parallel transport in the SVF setting. The rich theoretical background of SVF-based image registration provides effective and robust computational tools for the modelling of time-series of brain changes in neurodegenerative diseases. I will introduce statistical tools to analyse how the brain atrophy differs between different populations: beyond the usual local volume change (a scalar summary measure quantified by the Jacobian determinant of the transformation), the SVF setting allows constructing and comparing mean deformation trajectories that conserve all the information about the longitudinal (temporal) deformation. Thanks to these contributions I will finally propose innovative measures of anatomical changes aimed at precisely quantify longitudinal brain atrophy rates through the Helmholtz decomposition of diffeomorphic transformations.

C7: Tracts as distributions over trajectories: Tractography and Analysis

Aasa Feragen *(DIKU, Dep. of Computer Science, University of Copenhagen)*

Tractography is a family of algorithms that aim to estimate the trajectories of brain fibers from noisy diffusion weighted MRI data. The most typical approach for estimating such trajectories is fiber tracking, where the algorithm starts at a preselected seed point and keeps walking in an estimated "most likely" direction until a stopping criterion is reached. Many fiber tracking variants exist, some of which are probabilistic in the sense that each step is drawn randomly from a distribution of stepping directions. By sending many "walkers", such methods get a good estimate of the most likely trajectory in spite of the noisy data. However, tracking methods suffer from "path length dependency" which, probabilistically, results in a) propagation of uncertainty with distance to the seed point, and b) decrease in the probability of ever reaching a target point with its distance to the seed point, regardless of whether there is a physical connection or not.

In this talk we present a new and recently developed family of tractography algorithms which output a probability distribution over paths as an estimate of the fiber trajectories. The uncertainty of the distributions give a localized measure of tract uncertainty, which does not propagate with distance to the seed point and which does not suffer from path length dependency. The concrete algorithm returns tracts as Gaussian Processes that represent shortest paths in a random Riemannian manifold estimated from the data, in the special case of diffusion tensor imaging. However, the approach taken is general, and we will outline ongoing work aiming to apply the same approach to modern high angular resolution diffusion imaging data. Finally, we will discuss open problems regarding the analysis of the resulting tractography results.

C8: Practicalities and Challenges in diffusion MRI

Alexander Leemans *(ISI, University Medical Center Utrecht)*

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C9: Geometry in diffusion MRI - modeling and analysis

Tom Dela Haije *(CASA Eindhoven University of technology)*

Diffusion MRI is an imaging modality that can be used to measure the micron-scale displacements of diffusing water molecules in-vivo. A large part of the current research effort focuses on using such measurements as a means to characterize directional preferences in tissue, which makes it for example possible to recover maps of major neuronal connections in the human brain. In this talk I will focus on the role of geometry in both modeling, where the aggregate molecular dynamics are assumed to correspond to a simple stochastic (Brownian) process on a manifold, and in analysis, looking at the global geometrical properties of neuronal connections.

I will discuss the basic concepts of two common geometrical models used in diffusion MRI, in which the tissue is modeled as either a Riemannian or a Finslerian manifold. The Riemannian framework for diffusion MRI was originally proposed in 2002, and since then considerable effort has been made to extend it to the more complex Finslerian case. We recently introduced a canonical definition for the Finslerian geometrical structure in terms of the diffusion MRI signal, which solves one of the major hurdles in applying and understanding Finsler-based techniques.

After that I will present the results of a collaboration with Chantal Tax, where we looked at the global structure of neuronal fiber pathways in the brain. In recent work of Wedeen et al. these pathways were argued to form large scale sheet structures throughout the brain, whose presence we attempt to quantify by looking at the Lie bracket of the generating vector fields.