2DD50 - Exercises week 4

Theory: Kulkarni's book, sections 2.1-2.5: Handout, section 1

Conceptual Problems:

Conceptual problems 2.8, 2.9, 2.10, 2.11, 2.12 and 2.16 of Kulkarni's book. These can be found in the first part of paragraph 2.9.

Computational Problems:

The problems below refer to the computational problems in Kulkarni's book. These can be found in the second part of paragraph 2.9.

2.7 See the text in the book. To save some computational work, you may use that the matrix P in equation (2.12) of the book satisfies

$$P^3 = \begin{pmatrix} 0.91338 & 0.0857401 & 0.0008781 & 1.5 \times 10^{-6} \\ 0.0234499 & 0.947676 & 0.0287264 & 0.00014735 \\ 0.056897 & 0.0017577 & 0.926865 & 0.0144799 \\ 0.0288154 & 0.0008826 & 3 \times 10^{-6} & 0.970299 \end{pmatrix}$$

2.8 See the text in the book.

2.9
1. Explain the 1-step probabilities $p_{i,1}$ for $i = 0, 1, 2$ in the second column of the matrix P in formula (2.8) of Example 2.2.
2. Make the task as given in the text of this exercise in the book.

2.12
1. Explain in Example 2.4 the 1-step probabilities $p_{i,j}$ in the transition matrix P, with i and j the stock levels at 8.00 am. Monday in successive weeks.
2. Make the task as given in the text of this exercise in the book.

2.13 See the text in the book.

2.19 See the text in the book.

2.20 See the text in the book.

2.24
1. Give the set of balance equations and the normalizing equation for the DTMC in Computational Problem 2.20(d).
2. Make the task as given in the text of this exercise in the book *without* using a computer.

Extra exercises:

X1 Consider a Markov chain $\{X_n, n \geq 0\}$ with state space $S = \{1, 2, 3\}$ and transition probabilities $p_{12} = 1, p_{21} = 1/2, p_{23} = 1/2, p_{32} = 1/2$ and $p_{33} = 1/2$.

1. Compute $P(X_3 = j \mid X_0 = 1)$ for all j in S using Corollary 2.1 of Kulkarni's book.
2. Suppose that the Markov chain at some point transitions to state 2. What is the total expected time spent in that state until four transitions later?
3. Determine the limiting distribution of this Markov chain.

X2 A processor is inspected weekly in order to determine its condition. The condition of the processor can either be *perfect*, *good*, *reasonable*, or *bad*. A new processor is still perfect after one week with probability 0.7, with probability 0.2 the state is good, and with probability 0.1 it is reasonable. A processor in good condition is still good after one week with probability 0.6, reasonable with probability 0.2, and bad with probability 0.2. A processor in reasonable condition is still reasonable after one week with probability 0.5 and bad with probability 0.5. A bad processor must be repaired. The repair takes one week, after which the processor is again in perfect condition.

1. Formulate a Markov chain that describes the state of the machine, and draw the corresponding transition diagram.
2. Suppose that the processor is found to be perfect after an inspection. What is the probability that the processor is found to be good two inspections later?
3. Is this Markov chain irreducible? Is it periodic? If so, what is its period?
4. Determine the stationary distribution of the Markov chain.
5. Argue whether an occupancy distribution exists, and if so, compute it.
6. Argue whether a limiting distribution exists, and if so, compute it.

Handout section 1: Exercises 2,3 and 4.