2DD50 - Exercises week 5

Theory: Handout, section 1; Kulkarni's book, sections 2.6-2.7; Handout, section 2

Instruction: Handout section 1: Exercises 2,3 and 4

Conceptual Problems:

Conceptual problems 2.15, 2.18 and 2.19 of Kulkarni's book.

Computational Problems:

2.33 Consider the single machine production system of Conceptual Problem 2.10. Make the assumptions as given in the text of this exercise. To save computational work, you may assume that

$$P = \begin{pmatrix} 0.05 & 0.95 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 & 0.95 \\ 0.05 & 0.95 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 & 0.95 \\ 0 & 0 & 0.05 & 0.95 & 0 \end{pmatrix},$$

satisfies

$$P^4 = \begin{pmatrix} 0.0023 & 0.0862 & 0.0045 & 0.0857 & 0.8213 \\ 0.0002 & 0.0113 & 0.043 & 0.8168 & 0.1287 \\ 0.0023 & 0.0862 & 0.0045 & 0.0857 & 0.8213 \\ 0.0002 & 0.0113 & 0.043 & 0.8168 & 0.1287 \\ 0.0023 & 0.0862 & 0.0045 & 0.0857 & 0.8213 \end{pmatrix}, P^5 = \begin{pmatrix} 0.0003 & 0.015 & 0.0411 & 0.7802 & 0.1634 \\ 0.0022 & 0.0825 & 0.0064 & 0.1223 & 0.7866 \\ 0.0003 & 0.015 & 0.0411 & 0.7802 & 0.1634 \\ 0.0022 & 0.0825 & 0.0064 & 0.1223 & 0.7866 \\ 0.0003 & 0.015 & 0.0411 & 0.7802 & 0.1634 \end{pmatrix},$$

$$\sum_{i=0}^{9} P^i = \begin{pmatrix} 1.0604 & 1.3539 & 0.1829 & 3.4747 & 3.9281 \\ 0.0096 & 1.4261 & 0.1971 & 3.7453 & 4.6219 \\ 0.0604 & 1.3539 & 1.1829 & 3.4747 & 3.9281 \\ 0.0096 & 0.4261 & 0.1971 & 4.7453 & 4.6219 \\ 0.0104 & 0.4039 & 0.2329 & 4.4247 & 4.9281 \end{pmatrix}, \sum_{i=0}^{10} P^i = \begin{pmatrix} 1.0622 & 1.4225 & 0.1964 & 3.7317 & 4.5872 \\ 0.0103 & 1.455 & 0.2311 & 4.3908 & 4.9128 \\ 0.0622 & 1.4225 & 1.1964 & 3.7317 & 4.5872 \\ 0.0103 & 0.455 & 0.2311 & 5.3908 & 4.9128 \\ 0.0122 & 0.4725 & 0.2464 & 4.6817 & 5.5872 \end{pmatrix}.$$

a) Compute the expected number of items processed by the machine in the fifth minute, i.e., the expected number of items finished in that minute.

b) Compute the expected number of items processed by the machine in the fifth and sixth minute.

c) Make the task as given in the text of this exercise in the book.

2.37 For this problem, you may assume that the solutions to the equation $q = q * P$, with P as in Equation (2.10), are given by $q = d(0.3624, 0.3008, 0.1818, 1.1552)$ for all $d \in \mathbb{R}$.

a) Explain in Example 2.27 the formula for $c(i)$, the expected revenue per visit to state i ($i = 2, 3, 4, 5$), and verify the calculated values for $c(i)$.

b) Compute the expected revenues in the third week, starting at the beginning of the first week with the maximum stock.

c) Make the task as given in the text of this exercise in the book.

Extra exercises:

X3 Novak and Roger decide to play a game based on the Markov chain of exercise X1: $\{X_n, n \geq 0\}$ with state space $S = \{1, 2, 3\}$ and transition probabilities $p_{12} = 1, p_{21} = 1/2, p_{23} = 1/2, p_{32} = 1/2$ and $p_{33} = 1/2$. Every time the chain transitions to state 1, Novak pays Roger 20 euro. Likewise, if the chain moves to state 2, Roger gives 10 euro to Novak.

a) Suppose the chain is now in state 3. What is the expected amount of money Novak will have to pay to Roger during the following three transitions? What is the expected amount of money Roger loses over the course of these three transitions?

b) Who will be better off moneywise in the long run if Novak and Roger keep playing this game?

Serena has been observing Novak and Roger playing for a while. At some point, she decides that this is an utterly stupid game and that she would rather play some tennis. She demands that whenever the chain transitions to state 3, Novak and Roger stop playing this game and start to play a game of tennis against her instead.

c) Suppose that the chain is in state 2, when Serena gets her way. What is the expected number of transitions before a game of tennis is started?

X4 Consider the processor of exercise X2.

a) Whenever the processor is in a perfect or a good state, it processes 200 units of work per day. In a reasonable state, it only process 150 units of work in a day. Of course, the processor does not process anything if it is bad. What is the average number of work units that are being processed each week?

b) The repairman charges 500 euro every time he repairs the processor. How much money does the repairman earn on average from repairing this particular processor per year? You may assume that a year consists of 52 weeks.

c) What is the expected amount of time that passes by between two repairs?

Handout section 2: Exercises 1 and 2