2DD50 - Exercises week 6

Theory: Kulkarni's book, 4.1, 4.2 and 4.3

Conceptual Problems:

Conceptual problems 4.1 (assume $K = 2$), 4.2, 4.4, 4.5, 4.8 and 4.9 of Kulkarni's book.

Theory: Kulkarni's book, sections 4.6, 4.7.1 and 4.8

Computational Problems: (The problems refer to the computational problems in Kulkarni's book)

4.21
a) Give a set of balance equations (or rate-in-rate-out equations) and the normalising equation for the CTMC of Computational Problem 4.1.

b) Argue that the limiting distribution of this CTMC is given by

$$p = (0.2528,\ 0.1981,\ 0.2064,\ 0.1858,\ 0.1569).$$

4.26
a) Explain the expression $\mu_i = \min(i, M)\mu$ for the death rate from state i to state $(i-1)$ of the Call Center, as described in Example 4.10.

b) Give a set of balance equations and the normalising equation for the states of this Call Center with the data given in Computational Problem 4.9.

c) Assuming that p_i refers to the limiting probability that there are i callers in the system, verify that the limiting distribution $p = (p_0, p_1, \ldots, p_{12})$ of the CTMC corresponding to this problem is given by

$$p = (0.0023, 0.0140, 0.0421, 0.0843, 0.1264, 0.1517, 0.1517, 0.1300, 0.0975, 0.0731, 0.0548, 0.0411, 0.0309).$$

d) Compute the limiting probability that all reservation agents are idle.

e) Make the task as given in the text of this exercise in the book.

f) Calculate the fraction of time a reservation agent is busy.

g) Calculate the throughput, the average number of callers that is served per hour, of the Call Center.

4.27
a) Explain the expression $\mu_i = i/2$ for the death rate from state i to state $(i-1)$ of the Telephone Switch of Example 4.31.

b) Give a set of balance and normalising equations for the states of this Telephone Switch.

c) Compute the limiting probability that all lines in the switch are occupied.

d) Make the task as given in the text of this exercise in the book.

e) Calculate the fraction of time a line is occupied.

f) Calculate the throughput, the average number of calls that is served per minute, of the Telephone Switch.

4.38 See the text in the book.

4.44 See the text in the book.

4.45 a) Consider the CTMC of Computational Problem 4.1. Give the set of equations for the expected time to reach state 5, starting from the states 1, 2, 3, 4.

b) Make the task as given in the text of this exercise in the book.

Theory: Kulkarni's book, 5.2 and 5.3

Computational Problems:

Computational problems 5.1, 5.2, 5.3 and 5.9 of Kulkarni's book.