1 Instruction 1: Selection of exercises from Chapter 2 of Kulkarni with additional questions

Theory: paragraphs 2.1, 2.2, 2.3, 2.4

Conceptual Problems:

2.8 See the text in the book.

2.9 See the text in the book.

2.10 See the text in the book.

2.11 See the text in the book.

2.12 See the text in the book.

2.15 See the text in the book.

2.16 See the text in the book.

Computational Problems:

2.1
a) In Example 2.12, the number of new packets arriving at the switch in a time slot is assumed to be a Poisson random variable with mean 1. The buffer size is assumed to be $K = 7$. Explain how in the matrix P of this Example 2.12 (and of Example 2.8) the one-step transition probabilities $p_{5,j} = P(X_1 = j|X_0 = 5)$, from 5 packets in the buffer at time $n = 0$ to j ($j = 0, 1, \ldots, 7$) packets at time $n = 1$, can be determined from these assumptions and the further description of the switch.

b) Using the transition diagram of the DTMC of Example 2.12, describe the set of possible "paths" from state 7 (7 packets in the buffer) at time $n = 0$ to state 7 at time $n = 2$ and determine the probability $p^{(2)}_{7,7}$ of this set.

c) Using MAXIM, calculate $a^{(2)}_7$ given $a^{(0)}_7 = 1$. Explain why in this case the probability $a^{(2)}_7$ is equal to the probability $p^{(2)}_{7,7}$.

d) Make the task as given in the text of this exercise in the book.

2.5
a) In Example 2.8, the number of new packets at the switch in a time slot is assumed to have a bin(5, .2) distribution, a binomial distribution with parameters 5 and 0.2. The buffer size is assumed to be $K = 10$.
Explain in the 1-step transition probability matrix P of Example 2.8 the values of the probabilities $p_{i,j} = P(X_1 = j|X_0 = i)$, from i ($i = 1, 8$) packets in the buffer at time $n = 0$ to j ($j = 0, 1, \ldots, 10$) packets in the buffer at time $n = 1$.

b) Make the task as given in the text of this exercise in the book.

2.6
a) Let P be the transition probability matrix of the DTMC in Conceptual Problem 2.8 with $p = 0.95$. Calculate with MATLAB the 5-step transition probability matrix $P^{(5)}$. Use this matrix and the initial distribution given in the text of this exercise to calculate the transient pmf vector $a^{(5)}$ for time $n = 5$.

b) Draw a transition diagram of the DTMC in Conceptual Problem 2.8, again with $p = 0.95$. Describe the set A of possible "paths" from state 2 at time $n = 0$ to state 2 at time $n = 5$, calculate the probability of each path and the probability $P(A)$ of the set A.

c) The answer $P(A)$ from b) is equal to the probability $p_{2,2}^{(5)}$ in the 5-step transition probability matrix $P^{(5)}$ and equal to the probability $a_2^{(5)}$ in the transient pmf vector $a^{(5)}$, both calculated in a). Why?

d) Make the task as given in the text of this exercise in the book.

2.7 See the text in the book.

2.8 See the text in the book.

2.9 a) Explain the 1-step probabilities $p_{i,1}(i = 0, 1, 2)$ in the second column of the matrix P in formula (2.8) of Example 2.2.

b) Make the task as given in the text of this exercise in the book.

2.12 a) Explain in Example 2.4 the 1-step probabilities $p_{i,j}$ in the transition matrix P, with i and j the stock levels at 8.00 a.m. Monday in succesive weeks.

b) Make the task as given in the text of this exercise in the book.

2.13 See the text in the book.

2.16 See the text in the book. Use $M(51)$ instead of $M(52)$.