## 1 Instruction 7: Selection of exercises from Chapter 6 of Kulkarni and Handout, Section 4

## Theory: paragraph 6.7

## Computational Problems:

6.41 Consider the tandem queue with the data as described in the text of this exercise.
a) Make the tasks in this part of the exercise without using MAXIM. The results can be verified with MAXIM afterwards.
The network manager decides to station 3 servers at each station.

- Show that the limiting probability that there are $n_{1},\left(n_{1}=0,1, \ldots\right)$ customers at the first station is given by $p_{1}(0)=\frac{35}{187}, p_{1}(1)=\frac{56}{187}, p_{1}\left(n_{1}\right)=\left(\frac{224}{935}\right)\left(\frac{8}{15}\right)^{n_{1}-2} \quad n_{1}=$ $2,3, \ldots$
- The limiting probability that there are $n_{2}$ customers at the second station is given by $p_{2}(0)=\frac{5}{17}, p_{2}(1)=\frac{6}{17}, p_{2}\left(n_{2}\right)=\left(\frac{18}{85}\right)\left(\frac{2}{5}\right)^{n_{2}-2}$. Determine the simultaneous limiting probability $p\left(n_{1}, n_{2}\right)$ that there are $n_{1}$ customers at the first station and $n_{2}$ customers at the second station.
- Calculate for each station the mean number of customers and the mean waiting time of customers per visit.
Hint: Use the following series: $\sum_{i=2}^{\infty} i\left(\frac{8}{15}\right)^{i-2}=\frac{330}{49}$ and $\sum_{i=2}^{\infty} i\left(\frac{2}{5}\right)^{i-2}=\frac{40}{9}$.
- What is in the first and second station the mean number of occupied servers and the mean service time per visit of a customer.
- Calculate for each station the mean number of customers in the queue, waiting for a available server, and the mean queueing time per visit of a customer.
- Calculate the fraction of time a server in the first and second station is occupied.
- What is the throughput of the first and second station, i,e, the expected number of customers that leave a station per hour?
- Calculate the mean number of customers in the network and the mean waiting time of customers in the network.
- What fraction of the mean waiting time in the network a customer spends in the queue and what fraction of the mean waiting time a customer is served?
b) Make the task as given in the text of this exercise in the book.
6.45 Consider Jackson network 2 with the data as described in the text of this exercise. Make the tasks of the exercise without using MAXIM. The results can be verified with MAXIM afterwards.
a) Is the network stable?
b) Show that the limiting probability that there are $n_{2},\left(n_{2}=0,1, \ldots\right)$ customers at the second station is given by $p_{2}(0)=\frac{7}{12}, p_{2}\left(n_{2}\right)=\left(\frac{35}{114}\right)\left(\frac{5}{19}\right)^{n_{2}-1} \quad n_{2}=1,2, \ldots$.
c) The limiting probability that there are $n_{1}$ customers at the first station is given by $p_{1}(0)=0.1054, p_{1}(1)=0.2311, p_{1}(2)=0.2534, p_{1}\left(n_{1}\right)=(0.1853)(0.5482)^{n_{1}-3} \quad n_{1}=$ $3,4, \ldots$.
The limiting probability that there are $n_{3}$ customers at the third station is given by $p_{3}(0)=\frac{9}{29}, p_{3}\left(n_{3}\right)=\left(\frac{180}{551}\right)\left(\frac{10}{19}\right)^{n_{3}-1} \quad n_{3}=1,2, \ldots$.

Determine the simultaneous limiting probability $p(3,1,1)$ that there are 3 customers at the first station, 1 at the second station and 1 at the third station.
d) Calculate for each station the mean number of customers and the mean waiting time of customers per visit. Use the following series:
$\sum_{i=3}^{\infty} i(0.5482)^{i-3}=9.3257, \quad \sum_{i=1}^{\infty} i\left(\frac{5}{19}\right)^{i-1}=\frac{361}{196}, \quad \sum_{i=1}^{\infty} i\left(\frac{10}{19}\right)^{i-1}=\frac{361}{81}$.
e) Give for each station the mean number of occupied servers and the mean service time per visit of a customer (i.e. the mean time that a customer is served per visit)?
f) Calculate for each station the mean number of customers in the queue, waiting for a available server, and the mean queueing time per visit of a customer.
g) Determine the fraction of time a server is occupied in the first, second and third station.
h) What is the probability that all servers are occupied, in the first, the second and the third station, respectively?
i) Calculate the throughput of each of the three stations, the expected number of customers that leave a station per hour.
j) Determine the mean number of customers in the network and the mean waiting time of customers in the network.
k) What fraction of the mean waiting time in the network of an entering customer is spent in station 3?

1) What fraction of the mean waiting time in the network a customer spends in the queue and what fraction of the mean waiting time a customer is served?

## Theory: Handout, Section 4

## Exercises: 2 and 3

