P.I: The power supply unit of computer servers makes use of specialized power transistors that ensure the power output is very steady. Failure of the power unit is most often caused by the failure of these transistors. Due to physical reasons, the lifetime of these transistors is well modeled by a exponential distribution with unknown mean parameter $\mu > 0$, which has density

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-\frac{x}{\mu}} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} .$$

Let $X_1, \ldots X_n$ be independent random variables representing the lifetime of n different transistors (measured in days).

(a) Show that the maximum likelihood estimator of μ is given by the sample mean $\bar{X}$.

The manufacturer of the transistors claims these transistors last on average no less than $\mu_0 = 2000$ days. To check if this claim is reasonable you will test $H_0 : \mu = 2000$ against $H_1 : \mu < 2000$. Furthermore, you measured the lifetime of $n = 20$ transistors and computed the corresponding sample mean to be $\bar{x} = 1450$ days.

(b) From your knowledge of probability theory you know that $\frac{2n}{\mu}\bar{X}$ follows a χ^2 distribution with $2n$ degrees of freedom. Show that the test that rejects H_0 if

$$\bar{x} \leq \frac{\mu_0 \chi^2_{2n;1-\alpha}}{2n}$$

has type I error **exactly** α.

Hint: Begin by noting that

$$P\left(\frac{2n}{\mu}\bar{X} > \chi^2_{2n;1-\alpha}\right) = 1 - \alpha .$$

(c) Compute the p-value of the test in question (c) as accurately as you can, when $\bar{x} = 1450$ days. Should you reject the null hypothesis at significance level $\alpha = 0.1$?

(d) Suppose actually $\mu = 1300$. What is the power of the test in (b) in this case, taking $\alpha = 0.1$?