September 20, 2011

The following corrections are made after detailed questions in the process of writing [7] by Philippe Piret (e-mail: philippe.piret@crf.canon.fr):
“Reading notes on section 3 ... ,[6]” in February 2005.
A copy one can find at item [31] of [11].

I thank him cordially for drawing our attention to these mistakes.

Ruud Pellikaan

**Corrections and addenda of Section 3.2 on curves of type II**

1) Consider a polynomial of the form:

\[ X^a Y^c + Y^{b+c} + G(X,Y) \]

The assumption:

\[ \text{“} \deg(G) < b + c \text{”} \]

in Proposition 3.17 and Corollary 3.18 should be sharpened to

\[ \text{“} \operatorname{wdeg}(G) < a(b+c) \text{”} \]

where the weighted degree of \( X^a Y^\beta \) is \( \alpha b + \beta a \).
Notice that \( \operatorname{wdeg}(X^a Y^c) = \operatorname{wdeg}(Y^{b+c}) = a(b+c) \).

2) Clearly the condition \( \operatorname{wdeg}(G) < ab \) in case \( c = 0 \), implies \( \deg(G) < \min\{a, b\} \). This sharpened condition can be found in the papers [2,3]. On page 18 of the notes there is the question whether the assumption \( \operatorname{wdeg}(G) < ab \) as could be found in several Japanese papers, is allowed. This is indeed the case.

3) The following example shows that the assumption

\[ \deg(G) < b + c \text{ instead of } \operatorname{wdeg}(G) < a(b+c) \]

is certainly not enough. Consider for instance the polynomial:

\[ F = (Y^3 + X + 1)(Y^2 + X) = X^2 + Y^5 + G, \]
with $G = XY^3 + XY^2 + X + Y^2$. Then clearly $F$ is not irreducible and $F$ is of the form $X^a Y^c + Y^{b+c} + G$ with

$$c = 0, \ a = 2, \ b = 5, \ d = \deg_X(G) = 1 < a \text{ and } \deg(G) = 4 < 5 = b.$$ 

But $\text{wdeg}(G) = 1.5 + 3.2 = 11$ is not strictly smaller than $10 = ab$.

4) The proof in the notes in case $c = 0$ can without much effort adapted to the case $\text{wdeg}(G) < ab$ instead of $\deg(G) < \min\{a, b\}$. See instance (21) in the notes becomes:

$$\ldots ab - a\beta - ab = ab - \text{wdeg}(G) > 0.$$ 

5) Clearly something went wrong in the chapter [3]. As noticed, the argument on the uniqueness is not correct, but more importantly the argument on the existence is flawed also, since the starting of the induction argument concerning $\varepsilon$ on page 909 is not checked, which is exactly the statement that: $\text{wdeg}(G) < a(b + c)$.

6) On page 19 of the notes a notational irregularity in the paper [3] is observed. Sometimes it is $X^a Y^b + Y^{b+c} + G$ instead of $X^a Y^c + Y^{b+c} + G$. This is correct. It should always be: $X^a Y^c + Y^{b+c} + G$. In Example 3.16 of [3] it is claimed that the monomials

$$x^\alpha y^\beta, \ \alpha < a \text{ or } \beta < c$$

form a basis for $S$. This is indeed true, this can be checked by an induction argument, but it follows also from Gröbner basis theory. Since $X^a Y^c$ is the leading term of $F$ with respect to the weighted degree lexicographic order, a basis is obtained by the complement of all monomials of the form $X^\alpha Y^\beta$ with $\alpha \geq a$ and $\beta \geq c$.

9) Another nice proof of the irreducibility of a curve with equation:

$$X^a + Y^b + G(X, Y) \text{ with } \text{wdeg}(G) < ab$$

is given in Corollary 2.7 of the paper [1] and it is from [2, pp. 503]. It uses the fact that the corresponding Newton diagram is integrally indecomposable. This result is attributed to Stepanov-Schmidt [9, 10, 8].

References


