Error-correcting pairs for a public-key cryptosystem

I. Márquez-Corbella [1] R. Pellikaan [2]

[1] INRIA Rocquencourt - SECRET Team

[2] Dept. of Mathematics and Computing Science, Eindhoven University of Technology

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Public-key cryptography (PKC)

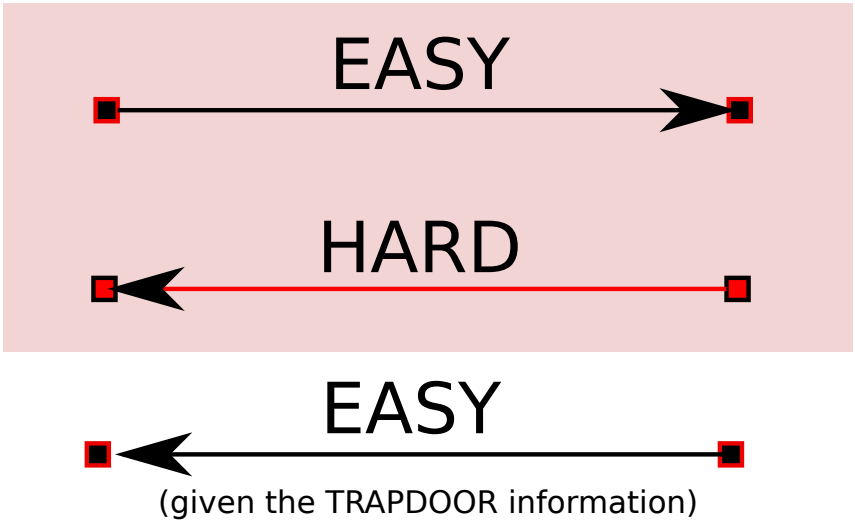
- Diffie and Hellman in 1976 in the public domain
- Ellis in 1970 for secret service, not made public until 1997
- advantage with respect to symmetric-key cryptography
- no exchange of secret key between sender and receiver

One-way function

- At the heart of any public-key cryptosystem is a
- one-way function
- a function $y = f(x)$ that is
- easy to evaluate but
- for which it is computationally infeasible, one hopes
- to find the inverse $x = f^{-1}(y)$

- Example
- differentiation a function is easy
- integrating a function is difficult

Public Key Cryptography



One-Way function

Trapdoor one-way function

Integer factorization

- $x = (p, q)$ is a pair of distinct prime numbers
- $y = pq$ is its product
- proposed by Cocks in 1973 in secret service
- Rivest-Shamir-Adleman (RSA) in 1978 in public domain
- based on the hardness of factorizing integers

Discrete logarithm

- G is a group (written multiplicatively)
- with $a \in G$ and x an integer
- $y = a^x$
- proposed by Williamson in 1974 in secret service
- Diffie-Hellman in 1974 and 1976 in public domain
- based on difficulty of finding discrete logarithms in a finite field

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➜ It can be attacked in polynomial time using **Shor's algorithm**

RSA

DSA

ECC

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Code-based Cryptography is a powerful alternative

Code based cryptography

- $h_1, \ldots, h_n$ is a given n-tuple of vectors in $\mathbb{F}_q^r$
- x is an n-tuple of elements in $\mathbb{F}_q$
- $y = \sum_{j=1}^{n} x_j h_j$
- proposed by McEliece in 1978
- based on the difficulty of decoding error-correcting codes
- it is NP complete

Trapdoor one-way functions - Decoder

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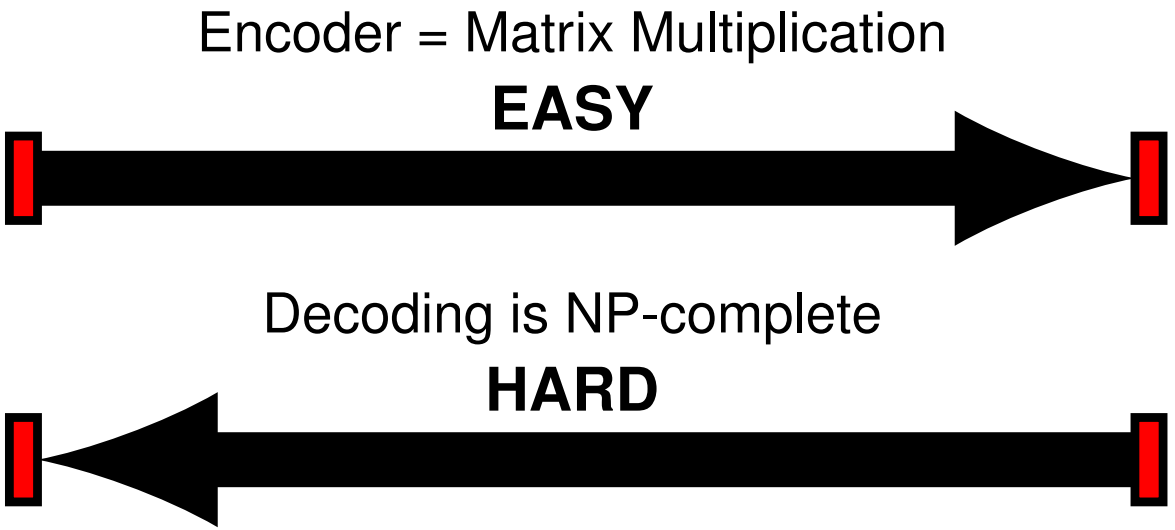

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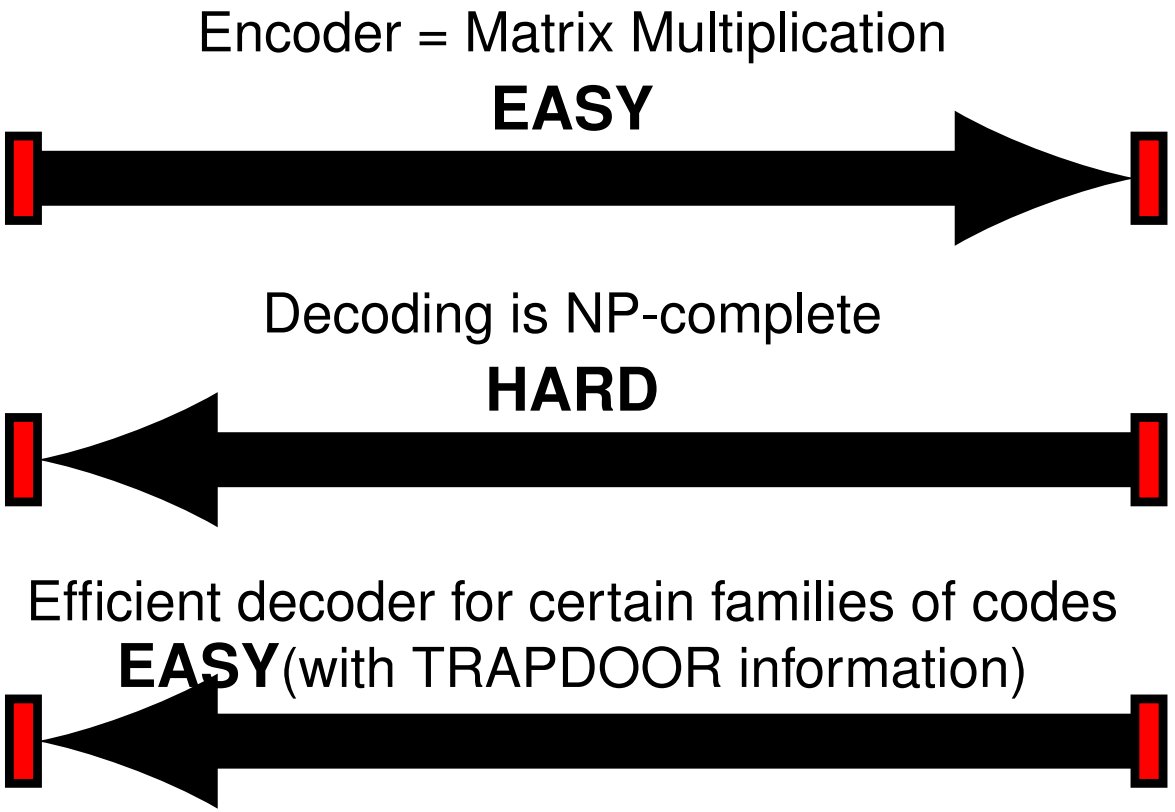


McEliece cryptosystem

➜ McEliece introduced the first PKC based on Error-Correcting Codes in 1978.

Advantages:

- Fast encryption (matrix-vector multiplication) and decryption functions.
- Interesting candidate for post-quantum cryptography.

Drawback:

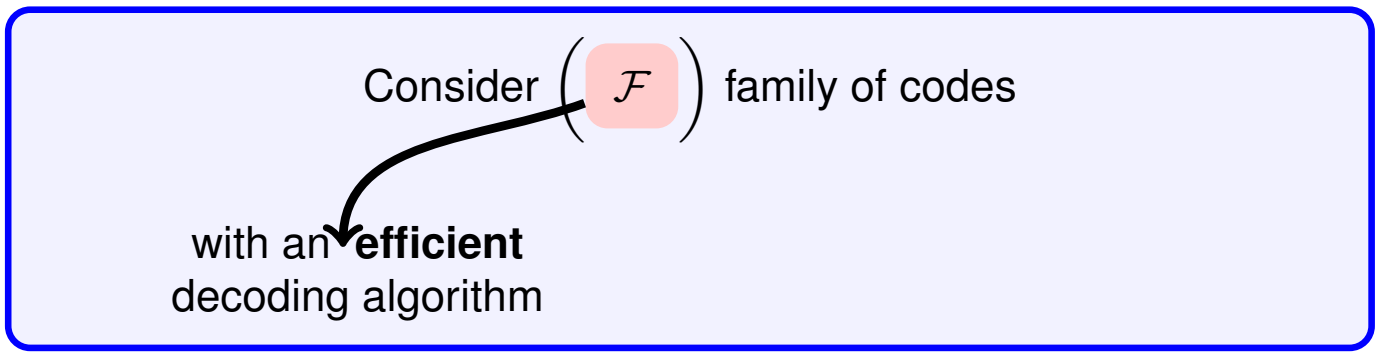
- Large key size.

R. J. McEliece.
A public-key cryptosystem based on algebraic coding theory.
DSN Progress Report, 42-44:114-116, 1978.

The McEliece Cryptosystem

Consider $\left(\mathcal{F} \right)$ family of codes

The McEliece Cryptosystem



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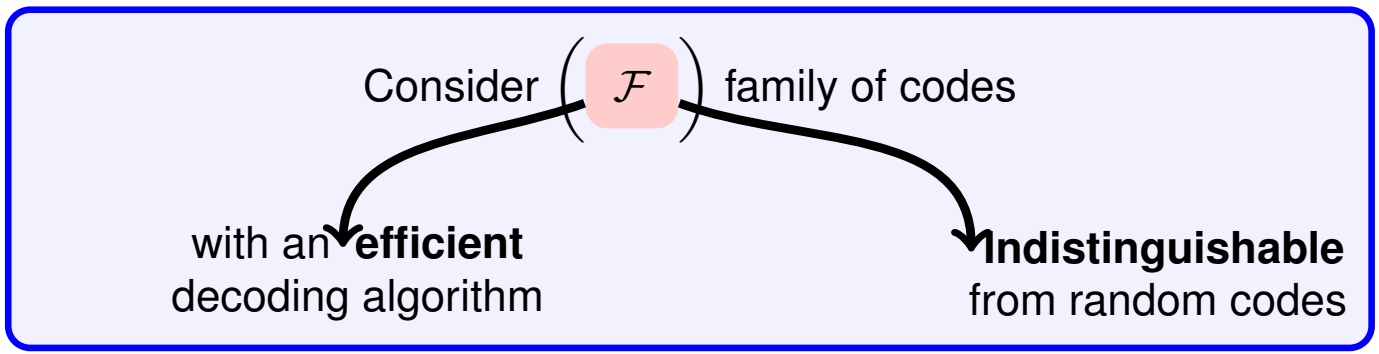

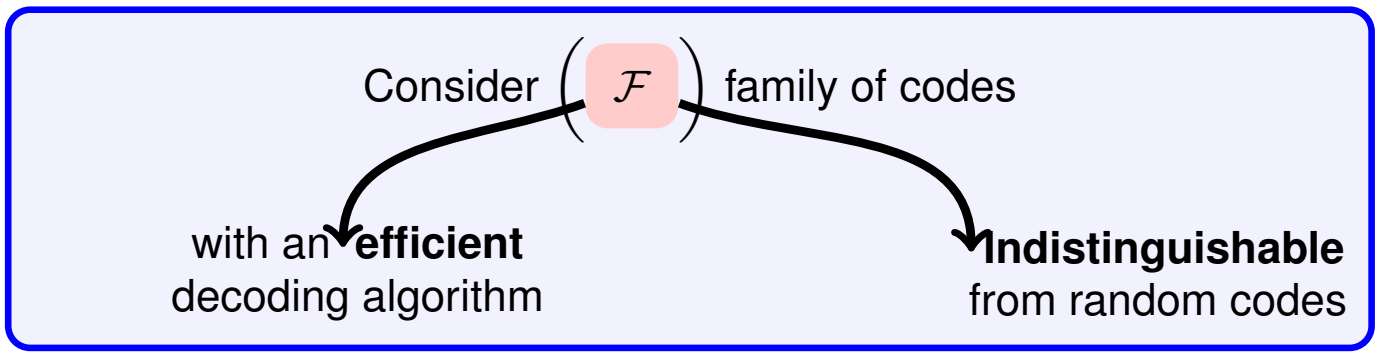

Key Generation Algorithm:

- $G \in \mathbb{F}_q^{k \times n}$ a **generator matrix** for $\mathcal{C} \in \mathcal{F}$
- $\mathcal{A}_{\mathcal{C}}$ an **"Efficient" decoding algorithm** for $\mathcal{C}$ which corrects up to t **errors**.

Public Key: $\mathcal{K}_{\text{pub}} = (G, t)$
Private Key: $\mathcal{K}_{\text{secret}} = (\mathcal{A}_{\mathcal{C}})$

Encryption Algorithm:

Encrypt a message $\mathbf{m} \in \mathbb{F}_q^k$ as

$$\text{ENCRYPT}(\mathbf{m}) = \mathbf{m}G + \mathbf{e} = \mathbf{y}$$

where $\mathbf{e}$ is a random error vector of weight at most t.

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Decryption Algorithm:

Using $\mathcal{K}_{secret}$, the receiver obtain $\mathbf{m}$.

$$\text{DECRYPT}(\mathbf{y}) = \mathcal{A}_{\mathcal{C}}(\mathbf{y}) = \mathbf{m}$$

Inner product and dual code

The inner product on $\mathbb{F}_q^n$ is defined by

$$\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + \cdots + x_n y_n$$

This inner product is bilinear, symmetric and non-degenerate

For an $[n, k]_q$ code C we define the dual or orthogonal code $C^\perp$ as

$$C^\perp = \{ \mathbf{x} \in \mathbb{F}_q^n \mid \mathbf{c} \cdot \mathbf{x} = 0 \text{ for all } \mathbf{c} \in C \}$$

Notation and prerequisites

- $\mathbb{F}_q$: Finite field with q elements.
- An $[n,k]_q$ **linear code** $\mathcal{C}$ over $\mathbb{F}_q$ is a k-dimensional subspace of $\mathbb{F}_q^n$
- The **Hamming weight** of $\mathbf{x} \in \mathbb{F}_q^n$ is $w_H(\mathbf{x})$.

Let $\mathcal{C}$ be a linear code over $\mathbb{F}_q$ we will denote by:

$n(\mathcal{C})$: Length , $k(\mathcal{C})$: Dimension and $d(\mathcal{C})$: Minimum distance

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MDS codes - Singleton bound

$$d(\mathcal{C}) \leq n(\mathcal{C}) - k(\mathcal{C}) + 1$$

If the equality holds $\Longrightarrow$ $\mathcal{C}$ is an **MDS code**

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Examples

1. The **zero code** of length n (i.e. the $[n, 0, n+1]$ linear code) and **its dual** (i.e. $\mathbb{F}_q^n$ which has parameters $[n, n, 1]$).
2. The $[n, 1, n]$ **repetition code** over $\mathbb{F}_q$
3. The **(Extended/Generalized) Reed-Solomon codes**

Star Product

Given two vectors $\mathbf{a} = (a_1, \ldots, a_n) \in \mathbb{F}_q^n$ and $\mathbf{b} = (b_1, \ldots, b_n) \in \mathbb{F}_q^n$ we denote by $\mathbf{a} * \mathbf{b}$ the **componentwise product**:

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Star Product of Codes

Let A and B be $\mathbb{F}_q$-codes of length n.
The **star product code** denoted by $A * B$ is:

$$A * B = \langle \{ \mathbf{a} * \mathbf{b} \mid \mathbf{a} \in A \text{ and } \mathbf{b} \in B \} \rangle$$

When $B = A$, then $A * A$ is called the **square** of A and is denoted by A^2

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- n, k nonnegative integers such that $1 \leq k \leq n \leq q$.

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Properties of GRS codes

Proposition: Parameters of GRS codes

The $\mathrm{GRS}_k(\mathbf{a}, \mathbf{b})$ is an $[n, k]_q$ code with minimum distance $d = n - k + 1$

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$$\mathcal{C} \text{ is } \mathbf{MDS} \iff \mathcal{C}^{\perp} \text{ is } \mathbf{MDS}$$

Proposition: The dual code of a GRS code is a GRS code

$$\mathrm{GRS}_k(\mathbf{a}, \mathbf{b})^{\perp} = \mathrm{GRS}_{n-k}(\mathbf{a}, \mathbf{b}')$$

Canonical Generator matrix for GRS

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$$
\begin{aligned}
G &= \begin{pmatrix} 1 & 1 & \ldots & 1 \\ a_1 & a_2 & \ldots & a_n \\ a_1^2 & a_2^2 & \ldots & a_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ a_1^{k-1} & a_2^{k-1} & \ldots & a_n^{k-1} \end{pmatrix} \begin{pmatrix} b_1 & & & 0 \\ & b_2 & & \\ & & \ddots & \\ 0 & & & b_n \end{pmatrix} \\
&= \begin{pmatrix} b_1 & b_2 & \ldots & b_n \\ b_1 a_1 & b_2 a_2 & \ldots & b_n a_n \\ b_1 a_1^2 & b_2 a_2^2 & \ldots & b_n a_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ b_1 a_1^{k-1} & b_2 a_2^{k-1} & \ldots & b_n a_n^{k-1} \end{pmatrix} = \begin{pmatrix} \mathbf{b} * \mathbf{1} \\ \mathbf{b} * \mathbf{a} \\ \mathbf{b} * \mathbf{a}^2 \\ \vdots \\ \mathbf{b} * \mathbf{a}^{k-1} \end{pmatrix} \in \mathbb{F}_q^{k \times n}
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Let:

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(A, B) is a ***t*-ECP** for $\mathcal{C}$ if the following properties hold:

- E.1 $(A * B) \perp \mathcal{C}$.
- E.2 $K(A) > t$.
- E.3 $d(B^{\perp}) > t$.
- E.4 $d(A) + d(\mathcal{C}) > n$.

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An $[n, k]_q$ code which has a t-ECP over $\mathbb{F}_{q^m}$ has an efficient decoding algorithm.

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An efficient decoding algorithm for GRS codes

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AN EFFICIENT DECODING ALGORITHM FOR GRS CODES

Let $\mathcal{C} = \mathrm{GRS}_k(\mathbf{c}, \mathbf{d}) \Longrightarrow \mathcal{C}^{\perp} = \mathrm{GRS}_{n-k}(\mathbf{c}, \mathbf{d}^{\perp})$

Consider the codes

$\mathcal{A} = \mathrm{GRS}_{t+1}(\mathbf{c}, \mathbf{d}^{\perp})$ and $\mathcal{B} = \mathrm{GRS}_t(\mathbf{c}, \mathbf{1})$

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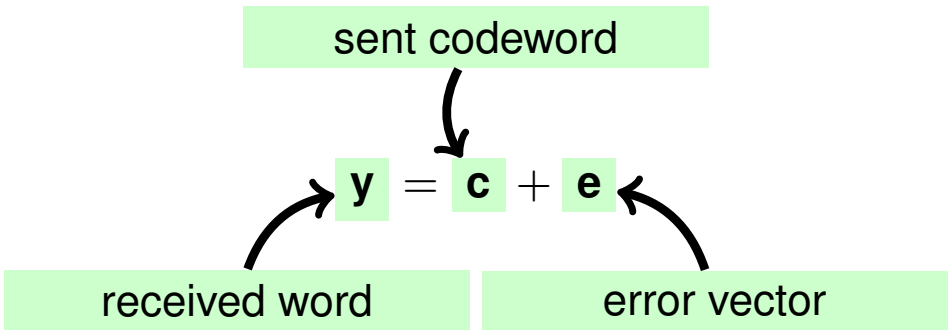

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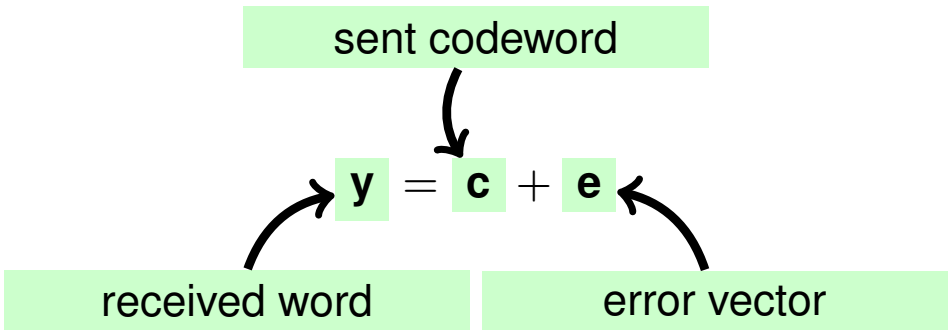

Define:

$$K_{\mathbf{y}} = \left\{ \mathbf{a} \in \mathcal{A} \mid \langle \mathbf{y}, \mathbf{a} * \mathbf{b} \rangle = 0 \text{ , for all } \mathbf{b} \in \mathcal{B} \right\}$$

An efficient decoding algorithm for GRS codes

$K_y = K_e$?

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Take notice that:

$$\mathcal{A} * \mathcal{B} \quad = \quad \mathrm{GRS}_{t+1}(\mathbf{c}, \mathbf{d}^{\perp}) * \mathrm{GRS}_{t}(\mathbf{c}, \mathbf{1})$$

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AN EFFICIENT DECODING ALGORITHM FOR GRS CODES

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If $w_H(\mathbf{e}) \leq t$, then $\mathbf{e}$ is a solution of:

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which **contradicts** the minimality of $d(\mathcal{C})$.

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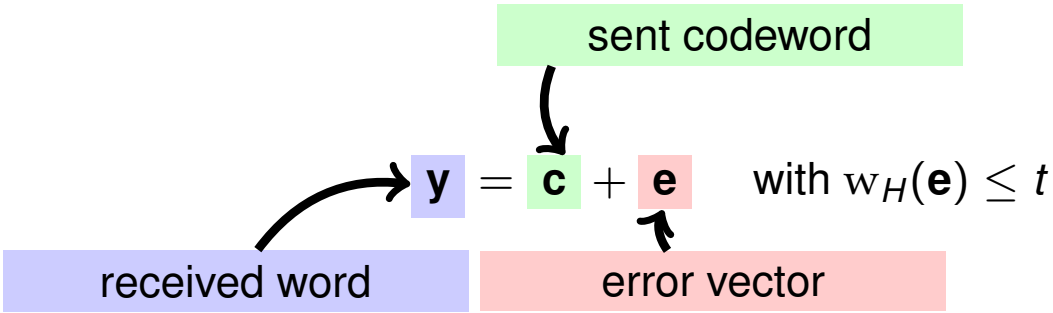
Error-correcting pairs (ECP)

Let $(\mathcal{A}, \mathcal{B})$ be a t-ECP for $\mathcal{C}$.



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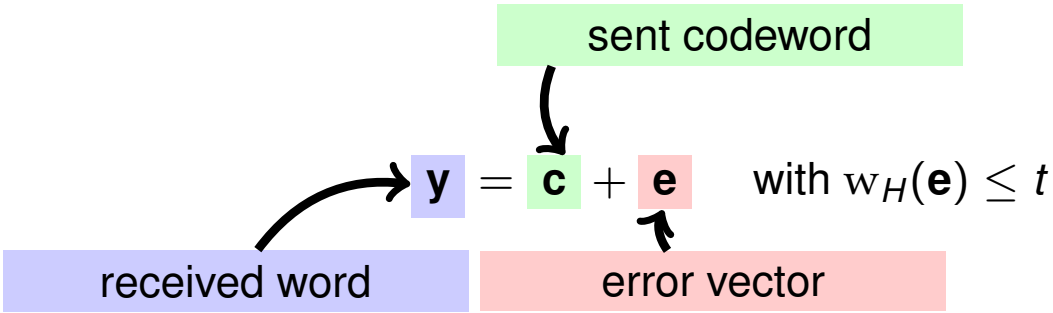

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$$\langle \mathbf{y}, \mathbf{a} * \mathbf{b} \rangle = 0 \text{ for all } \mathbf{b} \in \mathcal{B} \tag{1}$$

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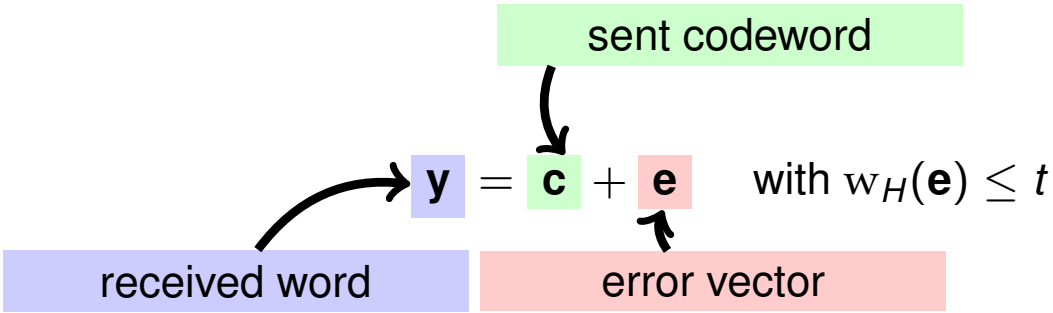

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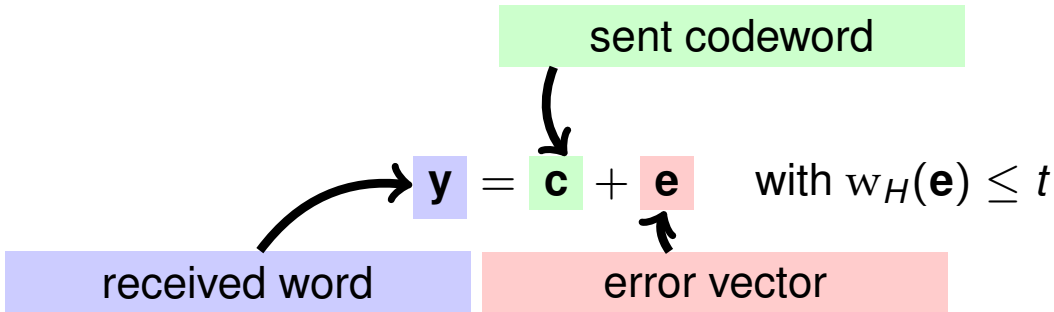

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Error-correcting pairs (ECP)

➜ t-ECP for Generalized Reed-Solomon (GRS) codes

I. Duursma

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I. Duursma, R. Kötter.

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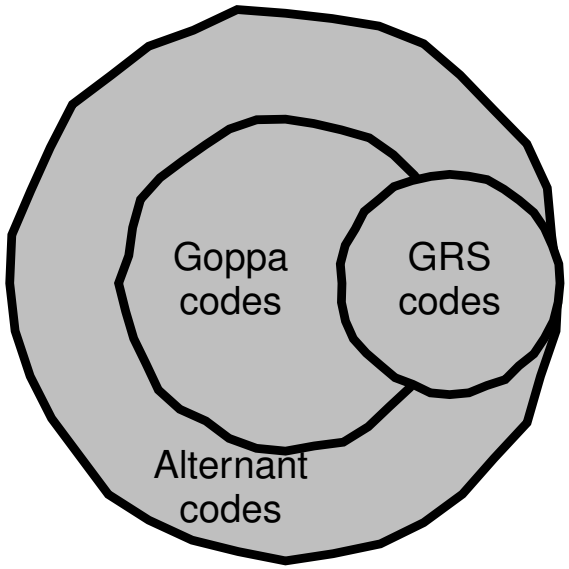

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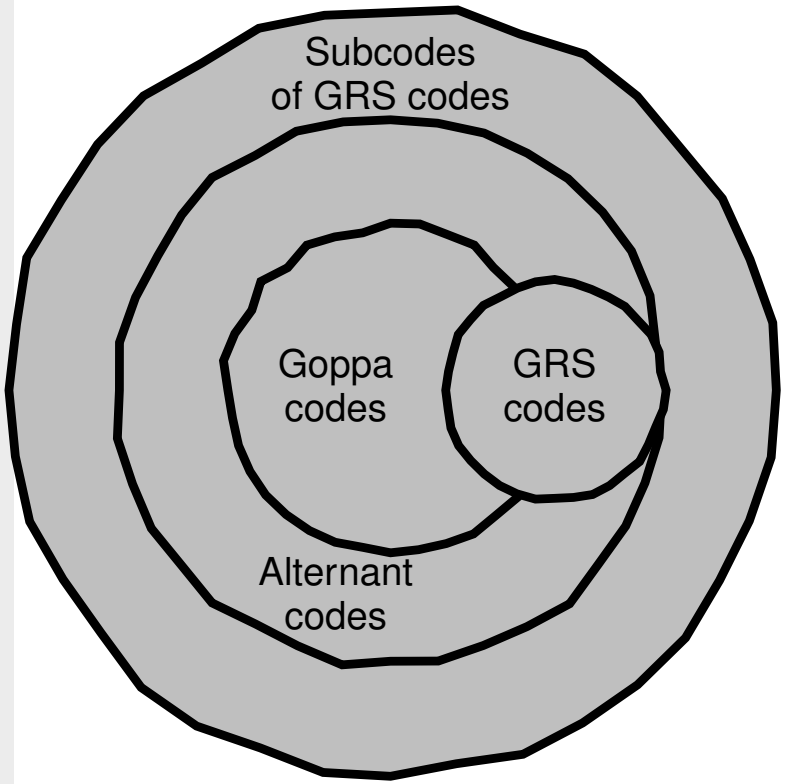

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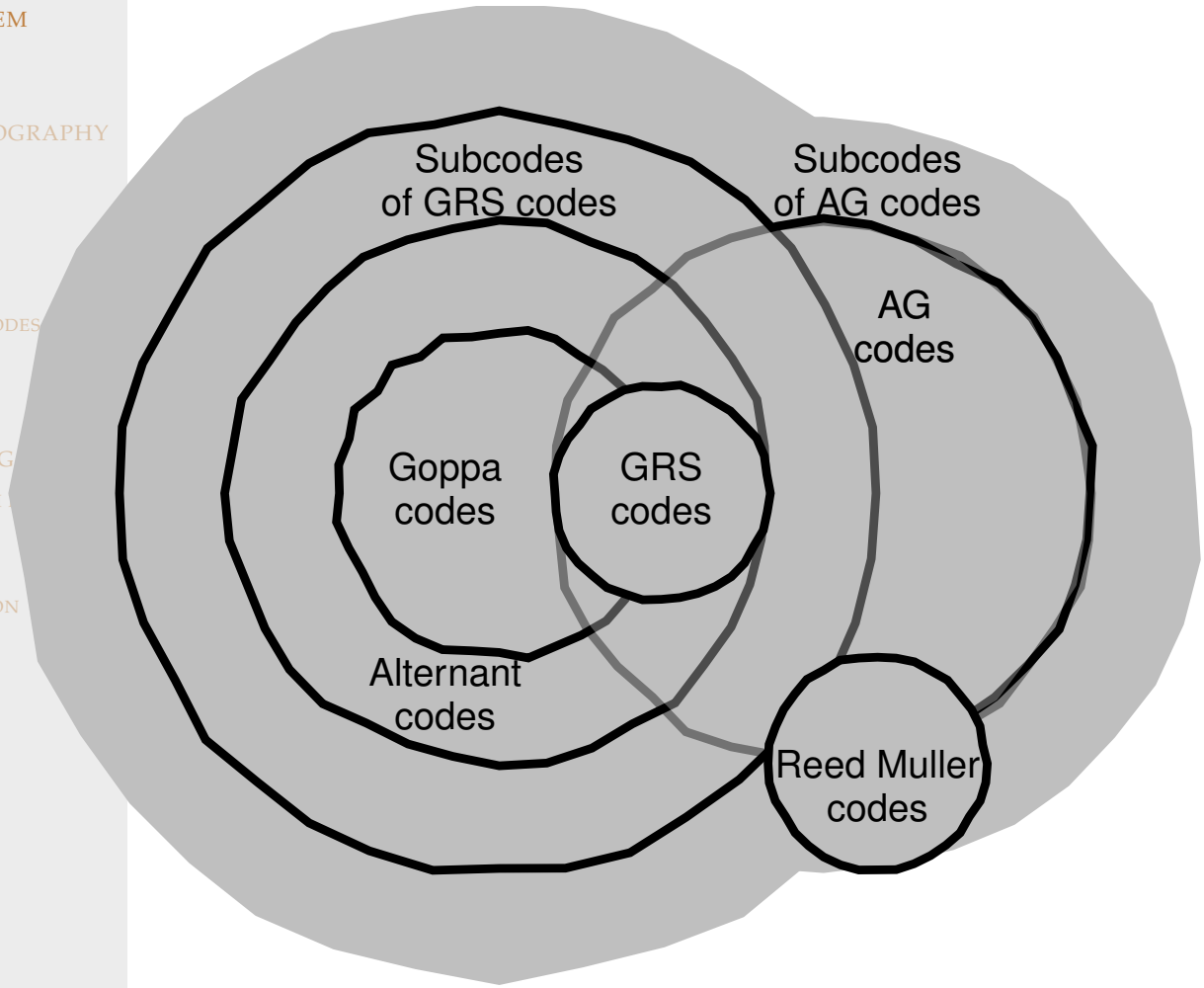

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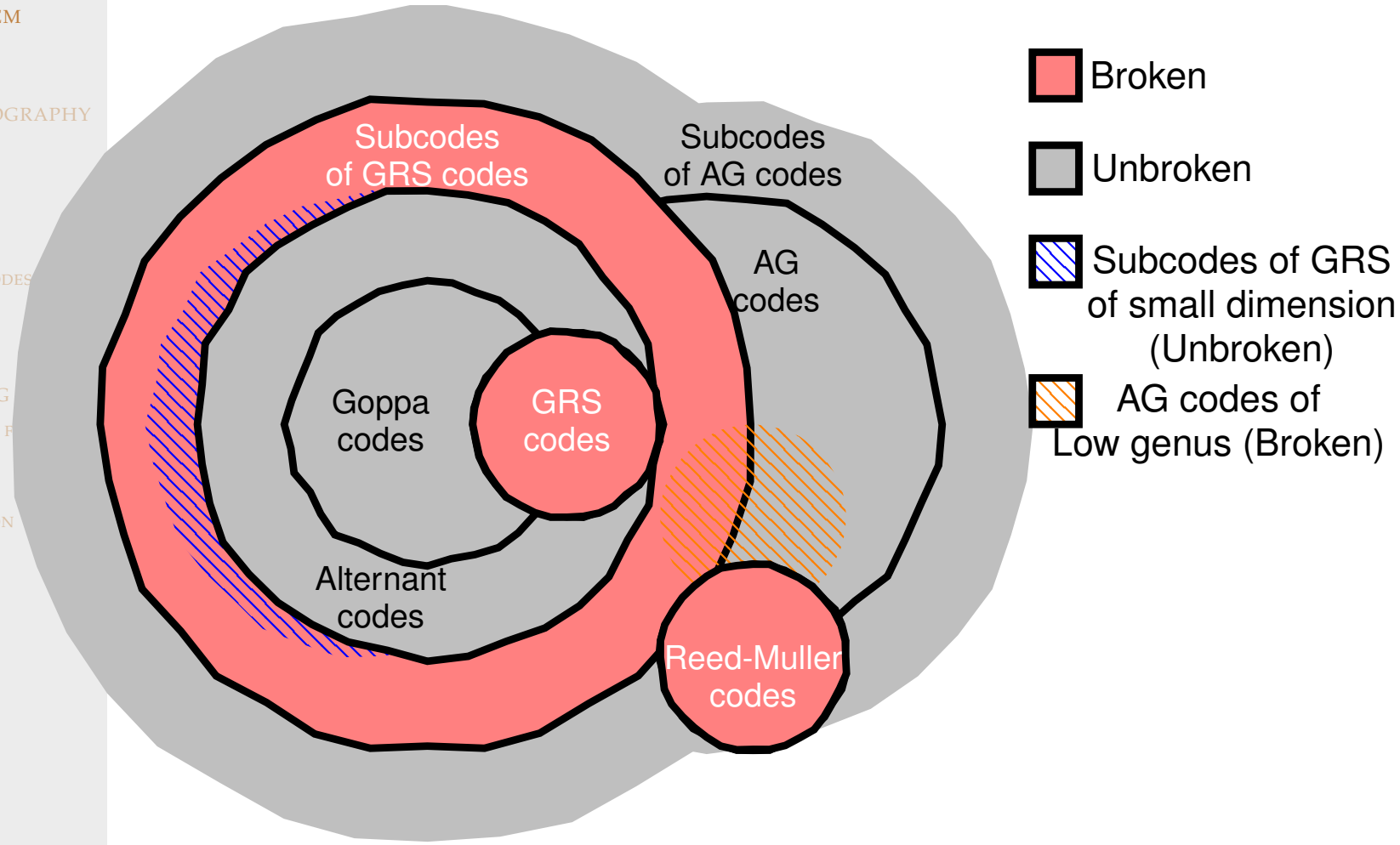

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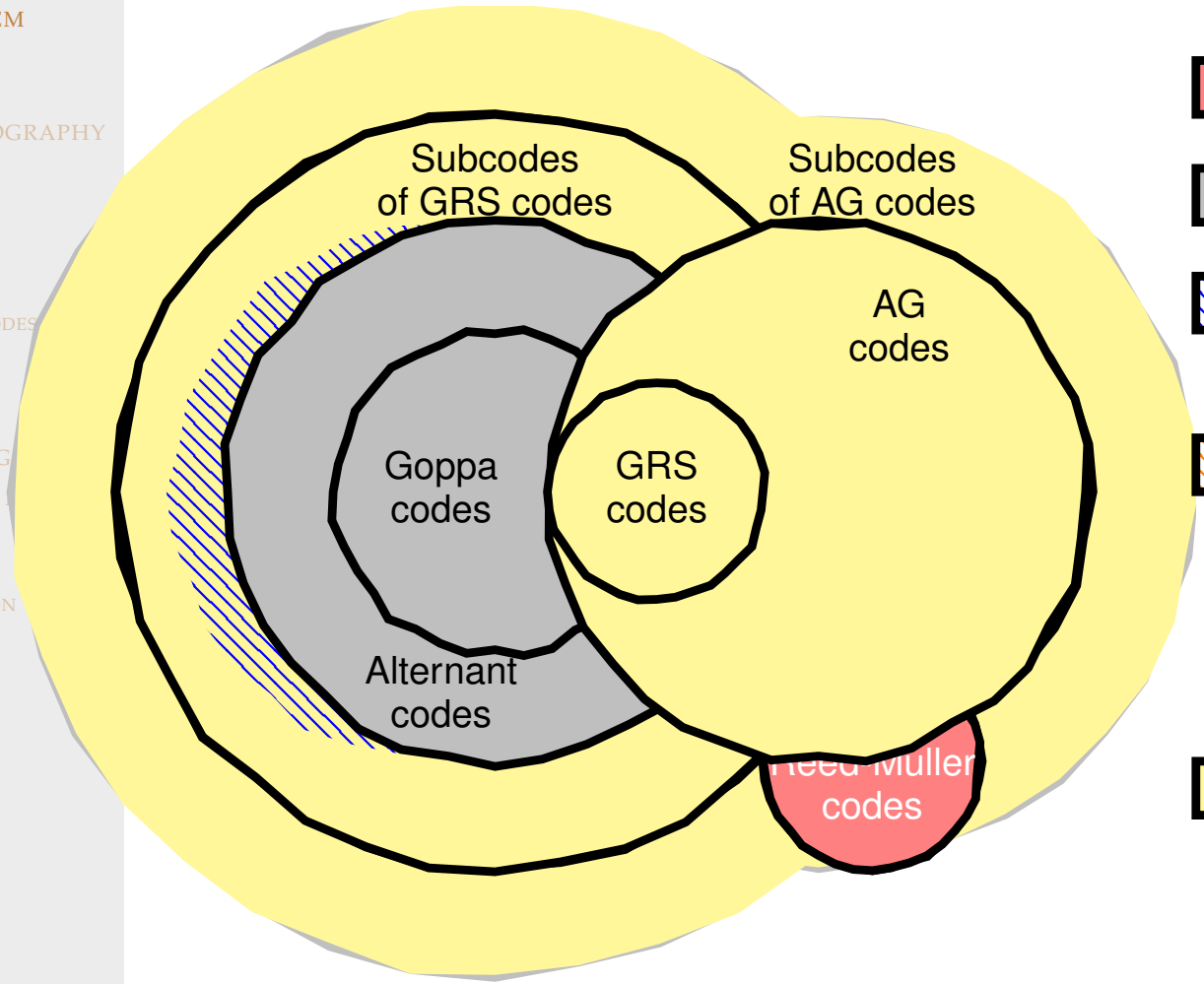


Broken

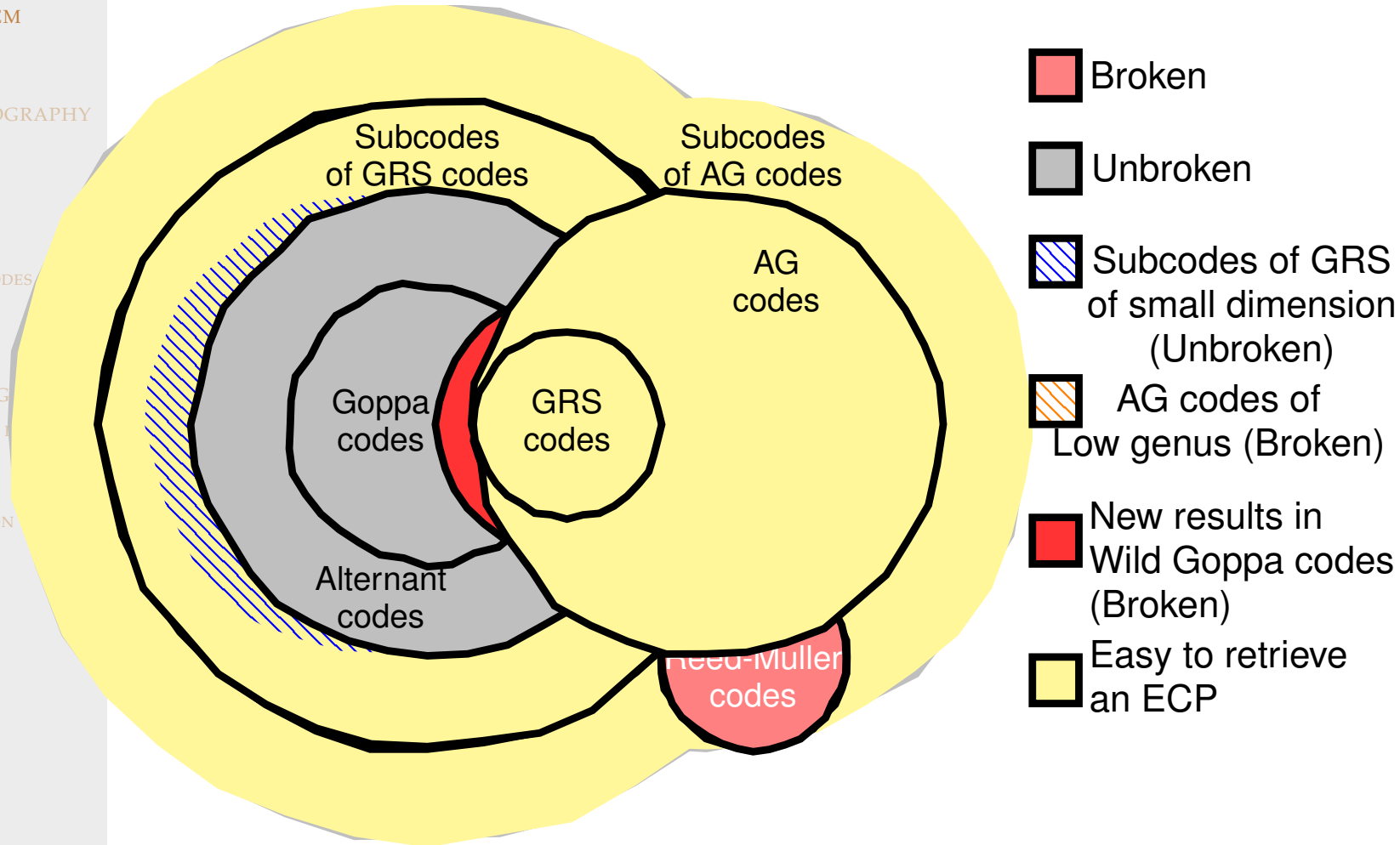
Unbroken

Subcodes of GRS of small dimension (Unbroken)

AG codes of Low genus (Broken)

Easy to retrieve an ECP

Codes with a t-ECP



ECP ONE-WAY FUNCTION

$\mathcal{P}(n, t, q)$ is the collection of pairs (A, B) that satisfy

$$\begin{array}{ll} E.2 \;\; k(A) > & E.5 \;\; d(A^{\perp}) > 1 \\ E.3 \;\; d(B^{\perp}) > t & E.6 \;\; d(A) + 2t > n \end{array}$$

Let

$$C = \mathbb{F}_q^n \cap (A * B)^{\perp}.$$

Then $d(C)$ is at least $2t + 1$ and (A, B) is a t-ECP for C

$\mathcal{F}(n, t, q)$ is the collection of $\mathbb{F}_q$-linear codes of length n and minimum distance $d \geq 2t + 1$

Consider the following map

$$\begin{array}{rccc} \varphi_{(n,t,q)} : & \mathcal{P}(n, t, q) & \longrightarrow & \mathcal{F}(n, t, q) \\ & (A, B) & \longmapsto & C \end{array}$$

The question is whether this map is a one-way function.

Conclusion

Main result

Many known classes of codes
that have have decoding algorithm correcting t-errors
have a t-ECP
and are not suitable for a code based PKC

Future Question: Is the ECP map a one-way function?

Thank you for your attention!